The Simplicity and Beauty of Topology: Connecting with the Intermediate Value Theorem

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Abstract

Topological concepts are foundational and provide structural support for ideas commonly investigated in high school mathematics and first semester calculus, particularly continuity and the Intermediate Value Theorem. Herein, we endeavor to introduce topological concepts to calculus teachers or to highly motivated students of calculus, and connect these concepts to the standard definition of continuity. We then introduce connectedness and prove the Intermediate Value Theorem. It is hoped that a topologically motivated proof will provide greater insight than the more commonplace epsilon–delta proofs associated with the Intermediate Value Theorem.

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