SML Round 1 2014-2015
Short Answers

1. **A** If x is the cost of the car, then $.9x = (540)(4)$ and so $x = (540)(4)/.9 = 2400$
2. **D** $s = mr + 1 = 1 - mr \Rightarrow mr = 0 \Rightarrow r = 0, s = 1 \Rightarrow r + s = 1$.
3. **B** The number of TV's per person $= \dfrac{2.5}{3.75} = \dfrac{2}{3}$
4. **D** Given the numbers listed, the answer cannot be A, B, or E. A quick check of the other two shows the answer is 53.
5. **D** $m + \dfrac{1}{m} = \dfrac{9}{20} \Rightarrow 20m^2 = 9m - 20 = 0 \Rightarrow m = \dfrac{5}{4}$ or $-\dfrac{4}{5}$. One is m and the other is M. Then $|M - m| = \dfrac{5}{4} - \left(-\dfrac{4}{5}\right) = \dfrac{41}{20}$.
6. **C** $\dfrac{a}{4} - \dfrac{b}{4} = \dfrac{a}{b} \Rightarrow ab - b^2 = 4a \Rightarrow (b-4)a = b^2$. So b must be greater than or equal to 5. A quick check of $b = 5$ gives $a = 25$, so the pair is $\left(\dfrac{25}{4}, \dfrac{5}{4}\right)$ with a sum of $\dfrac{30}{4} = \dfrac{15}{2}$.
7. **E** Let r_2, r_5, and r_{10} be the job rates on the copiers, in jobs per hour. Then $2.4(r_{10} + r_5) = 1(\text{job})$, $3(r_{10} + r_2) = 1$, and $4(r_5 + r_2) = 1$. This gives the system of equations:

   $r_5 + r_{10} = \dfrac{5}{12}$

   $r_2 + r_{10} = \dfrac{1}{3}$. Solving the system yields $r_2 = \dfrac{1}{12}$, $r_5 = \dfrac{1}{6}$, and $r_{10} = \dfrac{1}{4}$.

   $r_2 + r_5 = \dfrac{1}{4}$

   Letting x = the number of hours the machines can do the job together, we get $\left(\dfrac{1}{4} + \dfrac{1}{6} + \dfrac{1}{12}\right)x = 1$ or $x = 2$.
8. **A** You can see that A must be 1. Then T must be 8 or 9. Play a bit and you will see that T can't be 9, so T must be 8. Play a bit more to see that Y can be 4, 5, or 7.
9. **C** $18x^4 - 11x^2 + 1 = (9x^2 - 1)(2x^2 - 1) = 0 \Rightarrow x = \pm\dfrac{1}{3}, \pm\sqrt{\dfrac{1}{2}}$. So there are 2 irrational solutions.

10 **B** Let p be the probability of getting a green light. Then the probability of getting a red light is $\dfrac{p}{3}$. The probability of winning in the one push option is p. To win in the second option, you can get 2 green lights, with a probability of p^2, or a red light followed by a non-red color, with a probability of $\dfrac{p}{3}\left(1 - \dfrac{p}{3}\right)$, or a non-red color followed by a red light, also with a probability of $\dfrac{p}{3}\left(1 - \dfrac{p}{3}\right)$. So the probability of winning on the two push option is

$p^2+\frac{2p}{3}\left(1-\frac{p}{3}\right)=p^2+\frac{2p}{3}-\frac{2p^2}{9}=\frac{7p^2+6p}{9}$. For the options to have the same probability,

solve $p=\frac{7p^2+6p}{9}$ for p to get $p=\frac{3}{7}$.

11. **322** From knowing that $f(1)=3$, you get
$f(1)+f(1)=f(1)\cdot f(0)$ so $f(0)=2$. Then $f(2)+f(0)=f(1)\cdot f(1)$, so $f(2)=7$.
$f(3)+f(1)=f(2)\cdot f(1)$, so $f(3)=18$. $f(4)+f(0)=f(2)\cdot f(2)$, so $f(4)=47$.
$f(5)+f(1)=f(3)\cdot f(2)$, so $f(5)=123$. $f(6)+f(0)=f(3)\cdot f(3)$, so $f(6)=322$.

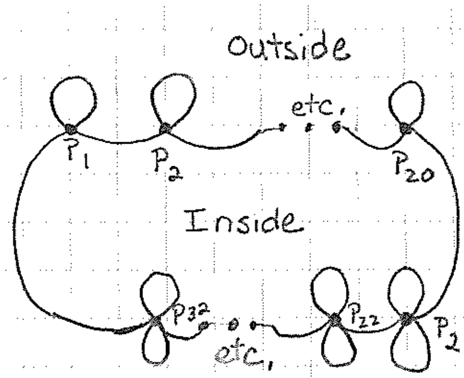
12. **A** $\sin 15^\circ\cdot 2\cos 15^\circ=\sin 30^\circ=\frac{1}{2}$. Bringing in the next term, you get

$\sin 30^\circ\cdot 2\cos 30^\circ=\sin 60^\circ=\frac{\sqrt{3}}{2}$. Continuing in this fashion, bringing in one factor at a time,

you get $\sin 120^\circ,\sin 240^\circ,\sin 480^\circ,\ldots,\sin 3840^\circ$. The last one has a value of $-\frac{\sqrt{3}}{2}$

13. **B** Do some serious number-crunching while changing the value of b to get that $a=3$ and $c=18$ when $b=31$ and the sum of these is 52.

14. **D** Points $P_1,P_2,\cdots,P_{20}$ each contribute one region inside their loops and points $P_{21},P_{22},\cdots,P_{32}$ each contribute two regions inside their loops, there is one region inside, and one region outside for a total of $20+2(12)+1+1=46$ regions:

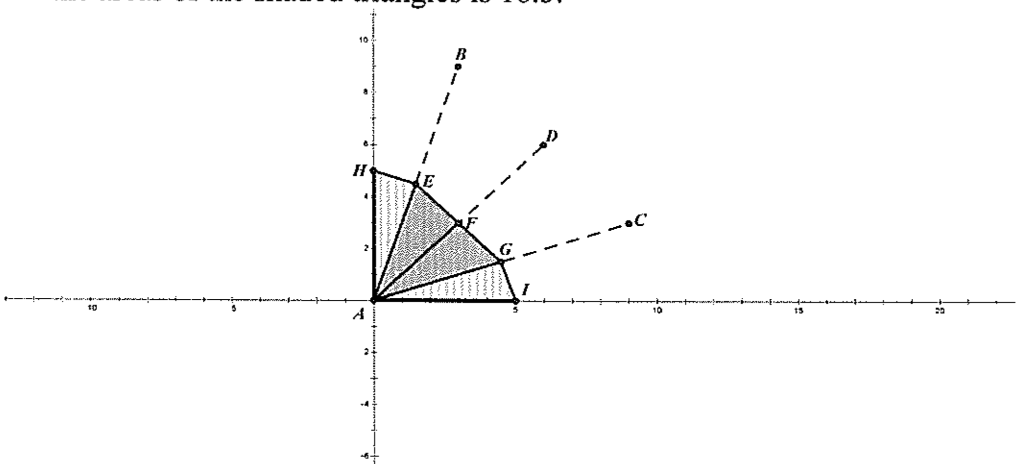


15. **C** Call a 2x2 subsquare made by 2 side by side dominoes a block. A covering of the 4x4 square could consist of 4 blocks. There are 2 ways to cover each block, so there are $2\cdot 2\cdot 2\cdot 2=16$ ways to cover the 4x4 square with four blocks. A covering could also consist of 3 blocks: Imagine two blocks on the left half and one in the middle of the right half – the blocks on the left can each be covered in 2 ways, the one on the right must be covered with vertical dominoes (if horizontal, there would be 4 blocks and we already counted those!) and the rest of the 4x4 is filled with two lone dominoes. The odd complete block can be on any of the four sides, so there are $4\cdot 2\cdot 2=16$ coverings of the 4x4 square with 3 blocks. To have exactly 2 blocks, you can go down the center band with horizontal dominoes (creating the two blocks) or across the center with vertical dominoes, for a total of 2 ways. Last, but not least, the 4x4 square can be covered with only 1 block of horizontal dominoes in the center (with horizontal dominoes above and below it and vertical dominoes to either side) or with 1

block of vertical dominoes in the center, giving 2 ways. Altogether, there are $16+16+2+2=36$ ways to cover the 4x4 square.

16. **E** I have to be honest – there are elegant mathematical ways to do this, but I just did a little number crunching. 1105 is divisible by 13 and subtracting various multiples of 5 from 110 gives a number divisible by 13 when the multiple is 9. Trying again with a larger number yields the same result.

17. **D** Let E, F, and G be the midpoints of AB, AD, and AC, respectively. The line through E that is perpendicular to AB has a y-intercept of 5. Label the y-intercept of that line as point H. In a similar way, the line through G that is perpendicular to AC has an x-intercept of 5. Label the x-intercept as point I. Points B, D, and C are collinear and thus points E, F, and G are collinear as well. All points in the shaded triangular regions shown below are closer to A than to any of the other points. Using the distances between those various points, the sum of the areas of the shaded triangles is 16.5.

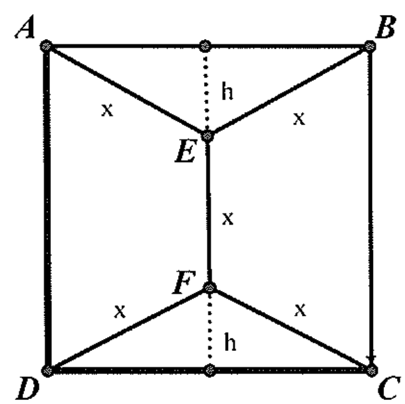


18. **B**

Area of $\triangle AEB = \frac{1}{2}(6)h = 3h = 3\sqrt{x^2 - 9}$ = Area of $\triangle DFC$

Area of trapezoid AEFD $= 3\left(\frac{x+6}{2}\right) = \frac{3x+18}{2}$ = Area of trapezoid BEFC

So, $6\sqrt{x^2-9} + 3x + 18 = 36$. Solving for x yields $x = 2\sqrt{7} - 2$



19. **A**

$$a_1 = 3$$
$$a_4 = a_3 + a_1 = a_3 + 3$$
$$a_5 = a_4 + a_2 = a_2 + a_3 + 3$$
$$a_6 = a_5 + a_3 = a_2 + 2a_3 + 3 = 30 \text{ so } a_2 + 2a_3 = 27$$
$$a_7 = a_6 + a_4 = 30 + a_3 + 3 = 33 + a_3$$
$$a_8 = a_7 + a_5 = 33 + a_3 + a_2 + a_3 + 3 = 36 + a_2 + 2a_3 = 36 + 27 = 63$$

20. **E** $P(x) = A\left(x^5 + \frac{B}{A}x^4 + \frac{C}{A}x^3 + \frac{C}{A}x^2 + \frac{B}{A}x + 1\right) = A \cdot Q(x)$ If $Q(x)$ factors as $(x-r_1)(x-r_2)(x-r_3)(x-r_4)(x-r_5)$, then $r_1 \cdot r_2 \cdot r_3 \cdot r_4 \cdot r_5 = -1$. Clearly, $Q(-1) = 0$, making -1 a solution of $P(x) = 0$. The remaining solutions must multiply to 1. Since $\sqrt{3} - 1$ is a solution, its reciprocal $\frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{2}$ is also a solution, making the correct answer to this question E.