SML Round 2 2014-2015
Short Answers

1. **B** 24 Euros $= p + .1p + .18p = 1.28p \Rightarrow p = 18.75$ so $18.75 \times 1.20 = 22.50$
2. **C** $2 + (n-1)x = 47 \Rightarrow (n-1)x = 45$. The largest value for *n* occurs when *x* is the smallest it can be. $x > 1 \Rightarrow x = 3 \Rightarrow n = 16$
3. **D** $6 - 4 = a$ and $2 + 8 = b \Rightarrow a + b = 12$
4. **C** $\frac{s_1 + s_2 + \cdots + s_5}{5} = 20 \Rightarrow s_1 + s_2 + \cdots + s_5 = 100 \Rightarrow \frac{100 + s_6}{6} = 24 \Rightarrow 100 + s_6 = 144 \Rightarrow s_6 = 44$
5. **D** $a_2 = 2a_1 - 1$ so a_2 is odd. $a_3 = \frac{a_2}{3}$ so a_2 must be 3 or 9. A quick check shows that $a_2 = 9$. So $a_3 = 3$ and $a_4 = 12 - 2a_3 = 6$.
6. **C** If K represents a knight and N represents a knave, the two possible configurations are KKNKKNKKN or KNKKNKKNK, each of which has 3 knaves in the circle.
7. **E** A little number crunching with A ÷ MA yields 6 ÷ 16 = .375. So A + T = 6 + 3 = 9.
8. **A** Use synthetic division to find $P(x) = Q(x)(x-1) + (a+b+c-1)$, Since $x - 1$ is a factor, $a + b + c - 1 = 0$. Do synthetic division to find that $Q(x) = R(x)(x-1) + (2a + b - 3)$. Since $x - 1$ is a factor of $Q(x)$, $2a + b - 3 = 0$. $\begin{matrix} a+b+c=1 \\ 2a+b=3 \end{matrix}$. Subtract the first equation from the second to find that $a - c = 2$.
9. **B** Rewrite as a rational function: $y = \frac{6x + 36}{x + 4}$. The vertical asymptote is the line $x = -4$ and the horizontal asymptote is the line $y = 6$. The point of symmetry is thus (-4,6). So $pq = -24$.
10. **A**
    Let $a_1 = 5, a_2 = 5r, a_3 = 5r^2, a_4 = 5r^2 + s, a_5 = 5r^2 + 2s, a_6 = (5r^2 + 2s)t,$ and $a_7 = (5r^2 + 2s)t^2 = 228$. Since $228 = 4 \cdot 57$, $t^2 = 4$ and thus $t = 2$ and $5r^2 + 2s = 57$. With some play, using $r = 3$ yields $s = 6$ and $a_4 = 51$.
11. **B** Because $12^3 = 1728 < 2015 < 13^3 = 2197$, then $a \leq 12$. If a = 12, then $b^3 + c^2 = 2015 - 1728 = 287$ and $b \leq 6$. No value of b from 1 to 6 is a solution to this equation. If a = 11, then $b^3 + c^2 = 2015 - 1331 = 684 = 8 + 676$, so a = 11, b = 2, and c = 26. Then a + b + c = 39.
12. **D** If $f(x) = kf(1-x)$ for all real numbers, then $f\left(\frac{1}{2}\right) = kf\left(1 - \frac{1}{2}\right) = kf\left(\frac{1}{2}\right)$, so k must equal 1.
13. **D** The question is equivalent to asking how many multiples of 3 from 3000 to 6045 have no even digits, which is equivalent to asking how many multiples of 3 from 3111 to 5997 have no even digits. Ignoring numbers with first digit 4 leaves numbers of the form 311x, 313x, 315x, …, 399x, 511x, 513x, …, 599x. The numbers of the form 311x, 317x, 335x, etc each can be followed by 1 or 7 to be divisible by 3, numbers of the form 313x, 319x, etc can be followed only by 5, and numbers of the form 315x, 333x, etc can be followed by 3 or 9. There are 8, 9, and 8 such sequences respectively, for a total of 2*8 + 9 + 2*8 = 41 such

numbers. A similar analysis with 5xxx shows 8 + 2*9 + 2*8 = 42 such numbers for a total of 41 + 42 = 83 such numbers.

14. **504** . At 6 pm there are 256 bacteria, so at 5 pm there must have been 0.5(256) + 256 = 384 bacteria. Similarly, at 4 pm there must have been 0.5(384) + 256 = 448 bacteria, at 3 pm 0.5(448) + 256 = 480 bacteria, at 2 pm 0.5(480) + 256 = 496 bacteria, and at 1 pm 0.5(496) + 256 = 504 bacteria.
15. **A** Since $y = 25 - x$, then $2\sqrt{25-x} = x - 1$ and $100 - 4x = x^2 - 2x + 1$. Then $x^2 + 2x - 99 = (x+11)(x-9) = 0$, and x = 9 or -11. But x = -11 does not satisfy the first equation, so the only solution is (9, 25 – 9), and b – a = 7.
16. **C** The total number of possible trips is $(4 \cdot 4 \cdot 3)^2 = 48^2$. Of these trips, $(4 \cdot 4 \cdot 3)(3 \cdot 3 \cdot 2) = 48 \cdot 18$ trips do not repeat a mode of travel. Thus the number of trips with a repeated mode of travel is $48^2 - 48 \cdot 18 = 48(48 - 18) = 1440$ trips.
17. **C** The sum of the first n positive integers is n(n + 1)/2. The sum of the 5 consecutive positive integers starting at a is 5a + 10, and the sum of the 8 consecutive positive integers starting at b is 8b + 28. Thus n(n + 1) = 10a + 20 = 16b + 56, and n(n + 1) must be divisible by both 10 and 8, which means it must be divisible by 40. The first such n is 15, with n(n + 1) = 240. But 240 = 16b + 56 means b = 11.5. The next such n is 24, with n(n + 1) = 600. This yields a = 58 and b = 34. Thus a – b = 24.
18. **E** It can be proven by mathematical induction that the positive fraction a/b is reduced to 0 in the least number of steps by replacing a/b with –b/a, then adding 1 repeatedly until the fraction first becomes positive, then repeating this process until a negative integer is reached. In the case of 7/9, this yields the sequence 7/9, =9/7, -2/7, 5/7, -7/5, -2/5, 3/5, -5/3, -2/3, 1/3, -3, -2, -1, 0. This requires 13 replacements.
19. **A** Label the quadrilateral ABCD, with AB = 10, BC = 4, CD = 6. Normally there would be 8 possible configurations satisfying the isosceles triangle requirement, but 3 of these are eliminated because they do not satisfy the Triangle Inequality. The possible configurations are then (1) AC = 10, AD = 6; (2) AC = 10, AD = 10; (3) BD = 6, AD = 6; (4) BD = 6, AD = 6; (5) BD = 4, AD = 10. Heron's Formula can be used to find the resulting areas, which are approximately (1) 36.179, (2) 60.068, (3) 31.583, (4) 56.472, (5) 30.821. Then A2 – A1 lies in [0,1].
20 **D** The sum of the elements of S = 95. Any multiple of 8, 12, or 24 always produces an even number, so T must contain either 0.5(22) or 1(29). In the first case, the remaining elements of T must add to 84. It can be shown by a variety of arguments that this is not possible. In the second case, the remaining elements of T must add to 66. Omitting 22, 12, or 8 can again be shown to be impossible. If 24 is omitted, 66 = 2(22) + 2(8) = 0.5(12), so the answer is 24.