SML Round 1 2013-2014
Short Answers

1. **A** If x is the price of an item, the discount is $\dfrac{.08x}{1.08x} \approx .07407$ or 7.4%

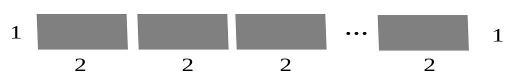
2. **E** The slopes of the lines are $-\dfrac{a}{2}$ and $\dfrac{b}{3}$. Since the lines are perpendicular, $-\dfrac{a}{2}\cdot\dfrac{b}{3} = -1,$ so $a \cdot b = 6.$

3. **E** Sue reduces the loan by \$190 each month. $12,000 - 190x = 0 \Rightarrow x = \dfrac{12,000}{190} \approx 63.158,$ so the loan will be paid off in 64 months.

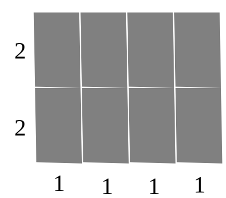
4. **A** $3x^2 + 4xy - 4y^2 = (3x-2y)(x+2y).$ The sum of the factors is $4x$.

5. **D** Solve the system $\begin{matrix} 2a+3b=6 \\ a+2b=5 \end{matrix}$ to get $(a,b) = (-3,4)$ so the sum is 1.

6. **D** The maximum perimeter is from all 8 dominoes being laid side by side:



So, $P = 16\cdot 2 + 1\cdot 2 = 34.$ The minimum perimeter is from the dominoes being stacked like:



So, $p = 4\cdot 2 + 8\cdot 1 = 16.$ Then $\dfrac{P}{p} = \dfrac{34}{16} = 2.125.$

7. **A** $217xy = 45a \Rightarrow a$ must be between $\dfrac{21700}{45}$ and $\dfrac{21799}{45}$. So a must be 483 or 484. $45\cdot 483 = 21735$ and $45\cdot 484 = 21780.$ In either case, $x + y = 8.$

8. **C** $x - \dfrac{1}{2}x = y - \dfrac{1}{3}y \Rightarrow x = \dfrac{4}{3}y.$ Since $x + y = 28,$ we get $y = 12$ and $x = 16.$ What remains is $(16-8)+(12-4) = 16$ ounces.

9. **D** $2^{60} - 1 = (2^{30}+1)(2^{30}-1) = (2^{30}+1)(2^{15}+1)(2^{15}-1).$ Using $x^3 + 1 = (x+1)(x^2 - x + 1)$ and $x^3 - 1 = (x-1)(x^2+x+1),$ we get

$2^{60} - 1 = (2^{10}+1)(2^{20} - 2^{10} + 1)(2^5+1)(2^{10} - 2^5 + 1)(2^5 - 1)(2^{10} + 2^5 + 1)$
$= 1025 \cdot 1047553 \cdot 33 \cdot 993 \cdot 31 \cdot 1057$

1025 is divisible by 5, 1047553 is divisible by 13, 33 is divisible by 11 (and by 3), 993 is divisible by 3, and 1057 is divisible by 7. None of the factors is divisible by 17, so only 5 of the elements of S are factors of $2^{60} - 1$.

10. **B** Let x be the number of 49¢ stamps and let y be the number of 3¢ stamps bought. Then $.49x+.03y=4.1$, or $49x+3y=410$. Do some number-crunching with $y=\dfrac{410-49x}{3}$ (making sure that x is larger than y) to find that $x=8$ and $y=6$, so $x+y=14$.

11. **B** A bit more number crunching! a^4 must be less than 2014, so $a\le 6$. On a graphing calculator, let $y=\sqrt{2014-x^2-a^2}$ where y represents c and x represents b. With $a=1$, look for whole number values of y in a table. There are none. Try with $a=2$ and so on until you find $a=3, b=42$, and $c=13$. Then $a+b+c=58$.

12. **E** Because bow and boy can be rolled, w and y must be on the same die and b and o are on different dice. Dry and red can be rolled, so e and y are on the same die and d and r are on different ones. So far, e,w, and y are on one die. Continuing in this way, since one can be rolled, o and n are on dice different from the one that has e, w, and y and thus won can be rolled.

13. **B** The answers provided are all in the 60's. $0.454545...=\dfrac{5}{11}$. A form of $\dfrac{5}{11}$ for which the numerator and denominator sum to a number in the 60's is $\dfrac{5}{11}\cdot\dfrac{4}{4}=\dfrac{20}{44}$ and $20+44=64$. Check that this works: $\dfrac{20}{44}\approx .455$ and $\dfrac{21}{45}\approx .467$ when rounded to 3 decimal places.

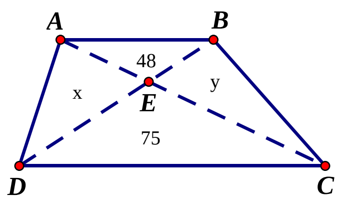
14. **C** If you solve the given system by eliminating x, you get $15b-a^2=225-ab$, or $b=\dfrac{a^2+225}{15+a}$. On a graphing calculator, let $y=\dfrac{x^2+225}{15+x}$ and use the table to find positive integer values for x between 1 and 30. Those are x (or a) $=3, 10, 15$, and 30 which add to 58.

15. **E** Elimination in the system leads to $v=4$. Substituting this into all equations gives:

$$3r+10s+16t+30u=-90$$
$$4r+15s+20t+36u=-133$$
$$5r+20s+24t+42u=-176$$
$$6r+25s+28t+48u=x-256$$

The coefficients in each column are arithmetic sequences. So is the last column, with the common difference being -43. So $-176-43=x-256$ or $x=37$.

16. **D** The area of trapezoid ABCD is $x+48+y+75$ as shown:



Using $\overline{AB}$ and $\overline{CD}$ as bases, traingles ABD and ACD have the same area. So $\dfrac{x+48}{x+75}=\dfrac{(1/2)\overline{AB}h}{(1/2)\overline{CD}h}=\dfrac{\overline{AB}}{\overline{CD}}$. Triangles ABE and CDE are similar, so $\dfrac{48}{75}=\left(\dfrac{\overline{AB}}{\overline{CD}}\right)^2$ so

$\frac{\overline{AB}}{\overline{CD}} = \frac{4}{5}$. So, $\frac{x+48}{x+75} = \frac{4}{5}$, or $x = 60$. Triangles *ACD* and *BCD* have the same base and height, so their areas are the same. So $x+75 = y+75 \Rightarrow y = 60$. So the area of the trapezoid is $60+48+60+75 = 243$.

17. **1471** $pqrs_7 = qrsp_9 \Rightarrow 343p+49q+7r+s = 729q+81r+9s+p \Rightarrow 171p = 340q+37r+4s$ $\Rightarrow 171p = 4(85q+s)+37r \Rightarrow r = \frac{171p-4A}{37}$ where $A = 85q+s$. Since these variables are used in a base 7 number, all must be between 0 and 6 inclusive. Let $y = r$ and let $x = p$. On a graphing calculator, let $y = \frac{171x-4A}{37}$ where, first, if $q = 1$, then $A = 85, 86, ..., 91$. Using the Table feature, look for integer values of *x* and *y* that are both between 0 and 6. There are none. Next, for $q = 2$, then $A = 170, 171, \ldots, 176$. We get integer values when $x = 4$ and $y = 0$ from $A = 171$ (so $q = 2$ and $s = 1$). So the number is $4201_7 = 2014_9 = 1471$.

18. **C** Suppose the candidates are A, B, C, D, E, F, and N (for "no one"). An example of your voting might be {A, A, A, B, B} or {A, B, C, N, N}. These are called multisets. The number of multisets of *r* items selected from a pool of *n* items is $\frac{(r+n-1)!}{r!(n-1)!}$. Here, that would be $\frac{(5+7-1)!}{5!(7-1)!} = 462$.

19. **D** Let *r* and *s* be the roots. Then
$P(x) = x^4 + mx^3 + nx^2 - 24x + 144 = (x-r)^2(x-s)^2$ or $(x-r)(x-s)^3$.
If $P(x) = (x-r)^2(x-s)^2$, then $P(x) = x^4 - 2(r+s)x + (r^2+4rs+s^2)x^2 - 2(rs^2+r^2s)x + r^2s^2$.
So $m = -2(r+s)$, $n = r^2+4rs+s^2$, $-24 = -2rs(r+s)$, and $144 = r^2s^2$. From the last of these two equations, $rs = \pm 12$ and thus $r+s = \pm 1$. The only factors of ± 12 whose sum is ± 1 are 3,-4 and -3,4. $r = -3$ and $s = 4$ don't satisfy $-24 = -2rs(r+s)$, but $r = 3$ and $s = -4$ do.
Then
$m = 2$ and $n = 9-48+16 = -23$. So $m+n = -21$.

20. **D**

n	Set	Odd-neighbored subsets of the set	Number
0	$\varnothing$	$\varnothing$	1
1	{1}	$\varnothing$, {1}	2
2	{1,2}	$\varnothing$, {1}, {1,2}	3
3	{1,2,3}	$\varnothing$, {1}, {3}, {1,3}, {1,2,3}	5
4	{1,2,3,4}	$\varnothing$, {1}, {3}, {1,3}, {3,4}, {1,2,3}, {1,3,4}, {1,2,3,4}	8

Notice that the number of odd-neighbored subsets is a Fibonacci number! Each subset in level *n* is either a subset from level *n* -1 or a subset from level *n* – 2 augmented with the numbers *n* and *n* – 1. For *n* = 12, the number of odd-neighborhood subsets would be 377, with 376 of them being nonempty.