1. Kara drives from San Francisco to Los Angeles in 8 hours. On the return trip (following exactly the same route), her average speed increases by 25%. The return trip takes 6 hours and how many minutes?

A. 20 B. 24 C. 30 D. 36 E. 40

2. For a natural number n, the radical of n is the largest square-free factor of n, and is denoted $rad(n)$. For example, $rad(8) = 2$ and $rad(252) = 42$. What is $rad(10!)$?

A. 105 B. 180 C. 210 D. 945 E. 990

3. There are three distinct digits x, y, z such that the decimal (base-10) number xyz is equal to the hexadecimal (base-16) number zyx. Find $x + y + z$ (in base-10). Note that xyz and zyx are <u>not</u> products here. For example, if $x = 2$, $y = 5$, and $z = 6$, then $xyz = 256$.

A. 11 B. 13 C. 15 D. 17 E. 19

4. A fair six-sided die is rolled four times in succession. What is the probability that the four successive outcomes are in strictly decreasing order (1st roll > 2nd roll > 3rd roll > 4th roll)?

A. 5/512 B. 5/432 C. 5/216 D. 11/512 E. 11/432

5. How many whole numbers n between 1 and 2000 inclusive satisfy <u>all</u> of the following four conditions: n is a multiple of 3, n is a multiple of 11, n is not a multiple of 5, n is not a multiple of 7?

A. 31 B. 32 C. 38 D. 41 E. 48

6. Between 10:52 AM and 3:18 PM on the same day, the hour hand of a standard wall clock sweeps out an area of 28 cm^2. Assuming that the hand moves at a constant velocity, the distance in cm traveled by the tip of the hour hand is closest to:

A. 11.1 B. 11.2 C. 11.3 D. 11.4 E. 11.5

7. The graph of the function $f(x) = \frac{-x^3+ax^2+9}{3x^2-bx}$ has an oblique (slant) asymptote that passes through (0, 13/9) and a vertical asymptote of $x = 2/3$. Find $a + b$.

A. 3 B. 5 C. 7 D. 9 E. 11

8. Whenever Vincent paints a wall, he paints the next wall in either half the amount of time or twice the amount of time as the previous wall (with equal probability). Find the expected number of minutes it would take Vincent to paint a room with four walls if he paints the first wall in 16 minutes and write it in the form p/q where p and q are relatively prime. What is $p + q$?

A. 126 B. 289 C. 373 D. 393 E. 473

9. An open box is made by cutting a N-inch by N-inch square from each corner of a 10-inch by 12-inch rectangular piece of material and then folding up the flaps. There are two different values of N that will result in a volume of 80 cubic inches. Find the sum of these two values of N, rounded to the nearest hundredth.

A. 2.76 B. 3.76 C. 7.24 D. 8.24 E. 10

10. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says something, but you could not hear it. Then Y says, “X said that he is the knave.” and Z says, “I am the knave.” Which of the following correctly identifies all three people?

A.	B.	C.	D.	E.
X is the knave	X is the spy	X is the spy	X is the knight	X is the knight
Y is the knight	Y is the knave	Y is the knight	Y is the spy	Y is the knave
Z is the spy	Z is the knight	Z is the knave	Z is the knave	Z is the spy

11. In ΔABC, AB = AC and in ΔDEF, DE = DF. AB is twice the length of DE and the measure of ∠D is twice the measure of ∠A. If a is the measure of ∠A, then the ratio of the area of ΔABC to the area of ΔDEF is given by:

A. 2 sec a　B. tan a　C. csc 2a　D. sec a tan a　E. sec 2a

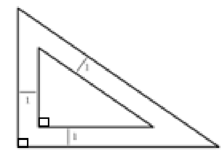
12. The solution to the equation $(\log_8 x^2)(\log_x 8)^2 = 1$ satisfies which inequality below?

A. $0 < x \leq 1$　B. $1 < x \leq 10$　C. $10 < x \leq 50$　D. $50 < x \leq 100$　E. $100 < x$

13. The number 2023 has two distinct prime factors, a and b. The number 2023! can be factored $2023! = a^m b^n q$ where q is a natural number not divisible by a or b. Determine $m + n$.

A. 55　B. 289　C. 408　D. 461　E. 477

14. In the figure on the right, the larger right triangle has sides with measures of 6, 8, and 10 units and the noted perpendicular distances between each pair of parallel sides are all 1 unit. Find the area of the smaller right triangle.



A. 4　B. 4.5　C. 6　D. 7.5　E. 8

15. Suppose a staircase has 10 stairs (starting at the bottom and stepping one stair at a time means it takes 10 steps to get to the top). Every time Leo takes a step, he either goes up a single stair, or skips a stair (so he goes up two stairs in one step), or skips two stairs (thus going up three stairs in one step). In how many different ways can he climb the staircase?

A. 55　B. 89　C. 156　D. 274　E. 548

16. Suppose 2023 congruent circles have centers which are equally spaced along the unit circle $x^2 + y^2 = 1$. Each of the 2023 circles is tangent to its two immediate neighboring circles. What is the radius of the circle which passes through every point of tangency?

A. $\cos\left(\frac{2\pi}{2023}\right)$　B. $\cos\left(\frac{\pi}{1012}\right)$　C. $\cos\left(\frac{\pi}{2023}\right)$　D. $\cos\left(\frac{\pi}{2024}\right)$　E. 1

17. A positive integer is called a "power length integer" if it has n digits and can be written in the form a^n for some positive integers a and n. For example, 125 is a power length integer since it has 3 digits and can be written as 5^3. There is a largest power length integer; find $a + n$ for that integer.

A. 21　B. 29　C. 30　D. 31　E. 36

18. Consider the following sums:

$$N = \sum_{i=0}^{100} \binom{100}{i} \qquad M = \sum_{i=0}^{100} i$$

What is the smallest positive integer k such that $M^k > N$?

A. 8　B. 9　C. 10　D. 11　E. 12

19. Consider the following five statements (which might be true or false) about a whole number N:

I. $10 \leq N \leq 20$　II. $5 \leq N \leq 15$　III. $2 \leq N \leq 12$　IV. N is odd or $N \leq 11$　V. N is a multiple of 3 or 5

Which of the five statements must be true whenever the other four are all true?

A. I　B. II　C. III　D. IV　E. V

20. In ΔABC, ∠ABC = 85.9°, AB has length 7, and the sum of the lengths of BC and AC is 20. Find the length of BC to the nearest tenth.

A. 8.5　B. 9.0　C. 9.5　D. 10.5　E. 11.0