

## Crossing Through and Bouncing Off $\infty$ : Graphing Rational Functions

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While rational functions are initially studied in high school, some of their behavior may be difficult to understand. For instance, when a rational function approaches a vertical asymptote, the graph may go to  $+\infty$  and then either return from  $+\infty$  or return from  $-\infty$ . But, where does it go and how does this happen? To explain this mathematical phenomenon, we consider topological techniques that wrap the real number line into a circle where  $-\infty$  and  $+\infty$  meet as one point. This then allows us to investigate principles of limits and local dominance in polynomial and rational functions. We also consider reciprocal relationships between zero and  $\infty$  and polynomial and rational parent functions. Altogether, we connect all of these notions to better understanding vertical asymptotic behavior of rational functions.

Keywords: rational functions, polynomial functions, vertical asymptotes, limits, topology, compactification



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