

Hands-On Multivariable Calculus: Activities with 3D-printed Models and CalcPlot3D Visualization

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Most students enter multivariable calculus with very little experience with the three-dimensional thinking they will encounter in this course. They have worked extensively in two dimensions, but the jump to three dimensions is challenging, as not all their skills and intuition transfer immediately to the new 3D setting. The concepts of multivariable calculus provide a rich environment for visualization and for making connections between symbolic computations and graphical representations. Although nearly everything in the course can be visualized, students are often willing to settle for mastering skill-based exercises and procedures with minimal conceptual understanding. Incorporating computer visualization and 3D models can help students go beyond these basic skills to develop an intuitive geometric understanding of the spatial concepts underlying the mathematical computations. This article presents a series of learning activities for multivariable calculus using 3D models and computer visualization that are designed to cultivate deeper conceptual understanding and to facilitate active learning in the classroom.

Keywords: multivariable calculus, visualization, 3D printing, active learning, tactile activities



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Paul Seeburger is a professor of mathematics at Monroe Community College, where he has taught since 1998. He earned an MA in mathematics from Central Michigan University. In addition to creating the CalcPlot3D app, Paul has led three NSF grant projects focused on helping students visualize multivariable calculus and differential equations.