

The Banach-Tarski Theorem

Recommended Reading

Much has been written about this theorem and searching the web yields many "hits". I've found most of the following articles and texts relatively easy to read and very informative. I've included comments.

1. Aczel, A., *The Mystery of the Aleph*, New York: Four Walls Eight Windows, 2000. (This is a fascinating biography of Georg Cantor.)
2. Banach, S., and A. Tarski, "Sur la decomposition des ensembles de points en parties respectivement congruents", *Fund. Math.* 6 (1924), 244 – 277. (This is the original publication of the theorem. The journal is Polish and the article is written in French.)
3. French, R., "The Banach-Tarski Theorem", *The Mathematical Intelligencer* 10 (1988), 21 – 28. (The article is an easy to read introduction to BT. The mathematics presented is minimal relative to other sources listed.)
4. Jech, T., *The Axiom of Choice*, Amsterdam: North-Holland, 1973. (Chapter 1 includes a brief discussion of AC, nonmeasurable sets, and a proof of BT.)
5. Reid, C., *Introduction to Higher Mathematics for the General Reader*, New York: Thomas Y. Crowell Company. (This includes an elementary description of AC and BT topics.)
6. Stromberg, K., "The Banach-Tarski Paradox", *Amer. Math. Monthly* 86 (1979), 151 – 161. (This is a wonderfully written elementary approach to BT. It is a full proof of the theorem with nothing omitted.)
7. Wagon, S., "A Hyperbolic Interpretation of the Banach-Tarski Paradox", *The Mathematica Journal* 3 (1993), 58 – 61. (The Hausdorff paradox is illustrated in the hyperbolic plane without the assumption of AC.)
8. Wagon, S., *The Banach-Tarski Paradox*, New York: Cambridge University Press, 1994. (This may be the definitive text about the theorem. The mathematics presented is much more sophisticated than the other sources listed here and, as a result, may be difficult to read. It includes an excellent list of references – over 250.)