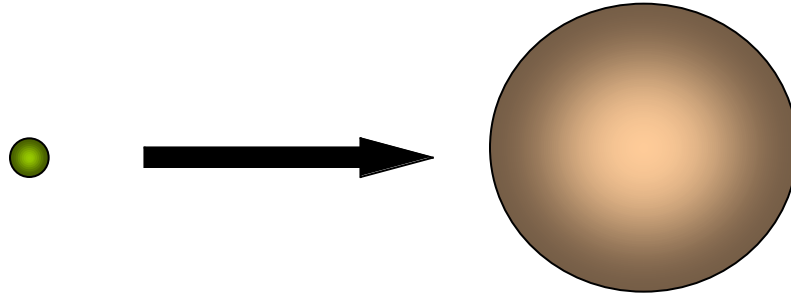


# A Top Down Look at the Banach-Tarski Paradox



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# The Banach –Tarski Paradox

In 1924, Stefan Banach (1892 – 1945) and Alfred Tarski (1901 - 1983) proved a remarkable theorem, known as the Banach-Tarski Theorem or the Banach-Tarski Paradox. Stan Wagon, author of [The Banach-Tarski Paradox](#), considers it “the most surprising result of theoretical mathematics”.

Formally –

“A closed ball can be decomposed into a finite number of pieces which, by rigid motions, can be reassembled to produce two balls, each having the same size and volume as the original.” The proof relies on the Axiom of Choice. An equivalent, and more whimsical, version of the theorem asserts that a ball of any size (a pea) may be decomposed into finitely many pieces and rearranged to form a ball of any other size (the sun). As a result, the theorem is sometimes referred to as the “pea and the sun” theorem.

Understandably, there was an uproar from both within and outside of the mathematics community. Some mathematicians argued that since the conclusion of the theorem was so absurd, the Axiom of Choice must be rejected. The majority, however, embraced the result, marveling at its counterintuitive nature.

Some individuals, outside of the mathematical world, complained that mathematicians were wasting their time on nonsense. An irate citizen once demanded of the Illinois legislature that they outlaw the teaching of this result in Illinois schools.

Most mathematicians today accept the Axiom of Choice and accept the conclusion of the Banach-Tarski Theorem. After all, the Axiom of Choice leads to interesting mathematics and, to reject it, would be just as disturbing as the conclusion of the Banach-Tarski Theorem.

What are we to make of such mathematical results? Do we dismiss them as nonsense and reject the mathematics which establishes such results, or do we open our minds to such bizarre and counterintuitive conclusions?

This presentation is an overview of the “BT” and its proof. For those interested in pursuing this topic, I strongly recommend the attached reading list. If I can be of assistance, please email me at [lwapner@elcamino.cc.ca.us](mailto:lwapner@elcamino.cc.ca.us).

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