

Integrating Liberal Arts Math and Environmental Science

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Seattle Central Community College

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Who we are:

Greg Langkamp has been a mathematics instructor at Seattle Central Community College for 12 years. He teaches courses ranging in levels from beginning algebra to vector calculus. Over the past few years his primary focus has been teaching math for liberal arts students and statistics.

Joe Hull has taught physical geology and environmental science at SCCC for 8 years, with a total of 18 years teaching in higher education. His geological research has focussed on the processes of mountain building. Joe is a councilor to the National Association of Geoscience Teachers.

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Our integrated course website:

<http://seattlecentral.org/faculty/jhull/env150.html>

Quantitative Environmental Learning Project (QELP):

<http://seattlecentral.org/qelp/>

Workshop Goals

Our goals for this workshop are the following:

- encourage participants to develop integrated math and environmental science activities and courses.
- provide insight into cooperative group learning activities, specifically those related to modeling and data collection.
- learn about the complexity of earthquakes and the statistics of their size distribution.

Food for thought ...

*We must free ourselves from seeing things the way they are!
A radical scientific view, indeed! If, following traditional scientific methods, we concentrate on an accurate description of the details, we lose perspective...*

As pointed out by Karl Popper, prediction is our best means of distinguishing science from pseudoscience. To predict the statistics of actual phenomena rather than the specific outcome is quite a legitimate and ordinary way of confronting theory with observations...

Thus, the problem of explaining the observed statistical features of complex systems can be phrased mathematically as the problem of explaining the underlying power laws, and more specifically, the values of the exponents.

Per Bak, How Nature Works

Our Integrated Course at Seattle Central Community College

For the last 4 years, we have taught an interdisciplinary course entitled 6 Billion People And Counting. This course integrates liberal arts math with introductory earth and environmental science. This course is fully integrated and blended, not just two classes stuck together on paper, with Math Guy lecturing one day and Env Guy lecturing the next. In this integrated class, Math Guy talks about population growth, and Env Guy helps students run exponential regressions on the calculator

This integrated course essentially acts as a "capstone" course for many students completing their Q requirement (college-level quantitative literacy) for the AA degree. It's a capstone course for many students who just need to finish their Q requirement before moving on. Non-science and non-math majors are the biggest audience so far, but more and more science majors are expressing interest, because of the opportunity to apply mathematics to the physical and natural worlds.

The main focus of this dual course is to apply mathematics to science, to put math and science together the way they are supposed to be. In this course, math is one of the languages of science, expressing the amount of lead in drinking water in quantitative terms. In this course, math is the tool of science, forecasting how often giant oil spills take place. In this course, math is used to understand environmental issues and problems, and help devise solutions that are based on reality, not appearance.

Instructors often find it difficult to integrate math and science courses because course curricula are jam-packed with topics that "must be covered". Though the situation is not much different at our community college, we have (fortunately) integrated two courses whose curricula are more flexible. Below is a table of the

environmental science and math topics that we cover, in chronological order as we teach them in our course.

**Environmental Science
(ENV 150)**

- Physical Earth
- Biota and Ecology
- Food and Soil
- Ecosystems and Biomes
- Human Population
- Water Resources
- Water Quality
- Waste and Recycling
- Energy
- Earthquakes and Hazards

**Liberal Arts Math
(Mat 107)**

- Units, Percents, and Displays of Data
- Linear Regression and Modeling
- Exponential Regression and Modeling
- Basic Statistics
- Difference Equations (linear, exponential)
- Difference Equations (affine, logistic)
- Systems of Difference Equations
- More Basic Statistics
- Normal Distributions
- Power Law Regression and Fractal Distributions

We use a fairly traditional environmental science textbook (Miller, G.T., *Sustaining the Earth, 4th Ed.*, Brooks Cole Pub). For mathematics readings and exercises we use a more obscure text (Schwartz, R.H., *Mathematics & Global Survival, 8th Ed.*, Simon and Schuster) as well as a comprehensive set of notes on difference equations. Students are required to either own or rent a TI-83 or TI-83+ calculator -- this is our principal computing tool for the course, although we also use Excel® (see below).

Students enroll in the following: 5 credits of MAT 107, 5 credits of ENV 150, 1 credit of MAT 299, 1 credit of ENV 299, and 1 credit of CSC101Q. With MAT 107, students fulfill their Q requirement for the AA degree. ENV 150 can be used as either a social science or a natural science. The two independent studies (299's) represent two one-day field trips for data gathering and instruction in field techniques. The CSC credit represents instruction in the use of Excel® spread sheet program in graphing, data analysis and modelling. Together, the courses give the student 13 credits of IS (integrated studies), 10 of which are required for the AA degree. Below is a typical weekly schedule for our course.

	M	T	W	Th	F
8 am	Weekly Quiz (30 min) + new week's overview	Lecture	Exercise	Lecture and Q&A	Lecture and Q&A
9 am		Lecture	Exercise	Lecture and Q&A	Exercise
10 am	Lecture	Lecture	Exercise	Lecture and Q&A	Exercise

**12
noon**

Excel Instruction

Excel Tutoring	Student tutor
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The course centers around a weekly group exercise (see Wednesday's and Friday's schedule above) where students solve or analyze environmental problems using real physical, chemical, and biological data. Sometimes the instructors provide the environmental data, sometimes the students find it on the web, and sometimes the students collect data themselves either in the lab or out in the field. Students work with real numbers and real

environmental examples, often messy and complex, not idealized or invented examples where all the points line up perfectly. A list of the student exercises we used in spring 2001 is given below.

1. Florida Lakes, Scale and Measurement.
2. Butter Clams and Linear Regression
3. Broiler Chickens and Exponential Regression

Field Trip #1 – NW Forests

4. Douglas Firs and Statistics
5. World Population and Difference Equations
6. Water Quality and Systems of Difference Equations

Field Trip #2 – NW Streams

7. Water Resources and Statistical Sampling
8. Hazardous Waste and Statistical Analysis
9. Energy Consumption and the Normal Distribution
10. Earthquakes and Power Law Distributions

These exercises are mostly data-driven, with a real environmental data set at the core of the exercise. The exercises are inquiry-based, with a large complement of thought questions in addition to the more "mechanical" tasks. We try to have the students constantly evaluate the mathematical analysis in physical terms; what are the units of slope, for example?

In this workshop we will be exploring Exercise #10 on earthquakes and power law distributions. To see Exercise #6, visit our website: <http://seattlecentral.org/qelp/NewBook.html> . If you

would like to site test any of the other exercises, contact us and we'll send you the entire collection.

As you can tell from the list above, there are two field trips incorporated into our course. These are all-day field trips on two different weekends, where students gather environmental data in small field teams. Students work with their own data during the following week. The first field trip visits old-growth, second growth and third growth forests just west of Mt. Rainier, to learn about forest ecology, tree biology, and logging practices. The second field trip visits a small stream in the Cascade foothills, where students measure stream profiles and collect data on stream discharge.

To round out the integrated course, students also learn the basics of Excel® to graph, analyze and model environmental information. Students complete 4 Excel® instructional modules which are spread throughout the quarter. Our Excel® modules cover the following 4 topics:

1. Creating Scatterplots
2. Cell Referencing
3. Cell Referencing and Difference Equations
4. Binning Data and Displaying Histograms

Accompanying each module is an Excel® homework problem which links to the environmental topic of the week. Additionally, students must submit one Excel® product (typically a graph) with their weekly group exercise.

Exercise #10: Earthquakes and Power Law Distributions

General Background:

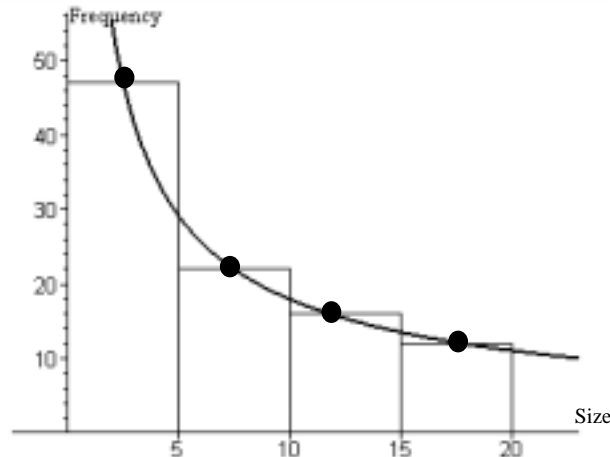
We present this student exercise on earthquakes with the hope that you can make use of it in some form in your courses. We recognize that you might not be team-teaching with an environmental scientist or geologist, and you might not be teaching a linked or interdisciplinary course. Whatever your teaching situation may be, we expect that there are some concepts, strategies, and ideas from this exercise that can be incorporated into your unique course setting. Finally, we note that the modified exercise that follows is parsed somewhat because (1) this is a workshop with time constraints and (2) you are mathematicians already. The complete student exercise follows this modified version.

The last week in our course is devoted to the study of geologic hazards and risk, and this exercise explores the frequency of occurrence of earthquakes of various sizes. A principal goal of this exercise is to help students understand the seemingly contradictory fact that individual earthquake events are not predictable, yet their overall statistical recurrence intervals can be quantified. This exercise is a bit different from many of our others in that students generate "data" from a model (the Bingo-Box, more on this later).

Mathematics Background: (you may not need this!)

The mathematics we use to model the size-frequency relationship of earthquakes, and many other naturally occurring phenomena, are power law distributions and power law functions. (For this reason, many of you may find this exercise appropriate for many courses outside of Liberal Arts Math including

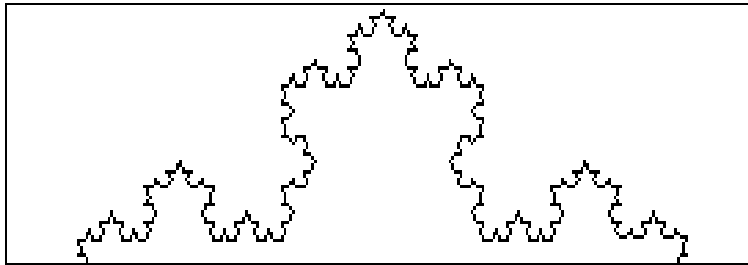
Precalculus and Statistics.) The connection between the frequency distributions and functions is simple: the frequency histogram (or relative frequency histogram) will have the general shape of a power law curve. See below.



The power functions we use are decreasing, i.e., of the form $f(x) = ax^{-k}$, with $k > 0$. Before our students work on this exercise, we have lectured for a few days of the general shape of power functions so that they understand the effects of the constants a and k . We also have explored the fact that data fit a power law model if and only if they appear linear when graphed in log-log space. In other words: A data point (x, y) will fit a power law function \Leftrightarrow the data point $(\log(x), \log(y))$ fits a linear function. Depending on the course in which you might employ this exercise, you will have to vary the level of student exposure to logarithms and the transformation of data in log-log space.

Data that fit a function of the form $f(x) = ax^{-k}$ are often said to have **fractal dimension equal to k** . In this exercise you'll explore the fractal dimension of bingo-box generated "earthquake" data. So what is the connection between fractals and earthquakes? Fractals, such as the coastline of England or the Koch Curve

(pictured below) have features, such as length, that depend on the scale of the ruler being used. The smaller the scale of measurement, the more length you can measure. When we graph length vs. scale, we get a decreasing power law function of the form $f(x) = ax^{-k}$.

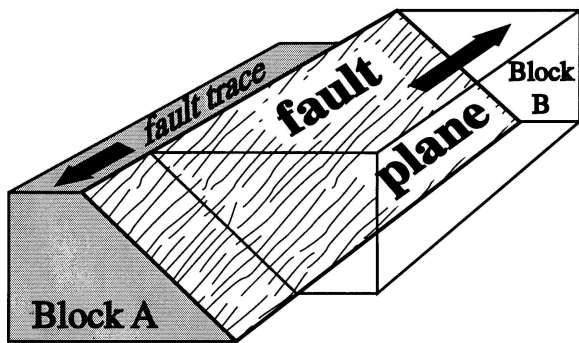


Another connection, although a bit more subtle, is that the faults that create earthquakes (see next section) are **space filling** just like fractals - again think of the Sierpinski triangle above. And as a matter of fact, the frequency of fault sizes show a power law distribution, just as do earthquake sizes. We're not experts on the subject of fractals, so don't ask too many tough questions! (Nonetheless, we hope you get as intrigued as we are!)

One final note: In this exercise, we use what might be called the "reverse cumulative frequency" of earthquakes, which is the number of earthquakes greater than or equal to an earthquake of a certain size. It's the "reverse" cumulative frequency because it's a decreasing sequence of values, starting with N, the total number. For brevity's sake, we refer to this sequence as the cumulative frequency in the exercise.

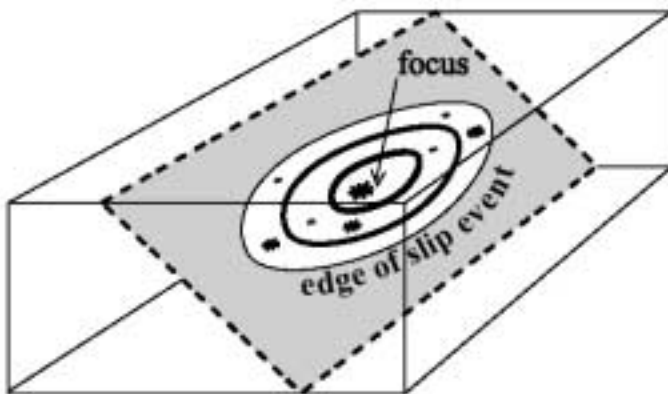
The Science of Faults and Earthquakes

Faults are approximately planar breaks in the Earth's crust and mantle. The media often refer to faults as "fault lines", but faults are planar features that extend down into the Earth. If faults intersect the ground surface (some do, some don't), then the line of intersection is called the **fault trace**. A fault is a break between adjoining **fault blocks**, which have moved relative to each other. There is no gap between the two blocks, just a planar feature (the fault) where rock has been pulverized and ground up during fault block movement.



A fault that has broken to the surface, with uniform sideways motion along the fault plane. Faults typically rupture along a small part of the fault during an event (see next diagram).

Faults can be very small, only a few square meters in area, or can be enormous, with lengths exceeding 1000 kilometers and surface areas up to 100,000 square kilometers. Faults do not extend forever; they can die out at their edges, if fault block movement decreases to zero, or they can intersect or join with other faults at their edges.



A single slip event, with contours of decreasing slip amount away from the focus. Main shock = big star, aftershock foci = smaller stars. Only an elliptical patch of the fault ruptures during a single event.

Active faults show active movement of one fault block relative to the other fault block. Geologists say that a fault is active if there has been relative motion across the fault during the last 10,000 years. The relative motion can be slow and constant (creeping motion), in which case no **earthquakes** (ground shaking) are produced. The relative motion can be fast and episodic, in which case earthquakes can be produced. Most active faults don't exhibit much fault block movement, then without warning, they move.

Fast motion along a fault produces earthquakes; this is a cause and effect relationship. The motion of the fault blocks and the shaking of the ground are two different things. As the two fault blocks slide past each other, vibrations or waves are produced, which radiate out in all directions away from the fault and fault blocks. There are several types of **seismic waves** or vibrations that are created by fast fault block movement. One of these types of waves travels along the surface of the Earth; it's these **surface waves** that people typically feel and that cause damage.

The more fault block movement in a single event, the bigger the resulting ground shaking. There is no direct relationship between the movement and shaking, but we can look at a couple of pairs of fault movements and earthquakes to see how the two compare. For example, a fault that moved a centimeter or so in 10 seconds might produce an earthquake with a Richter magnitude of 4 to 5 or so. For example, a fault that moved 2 meters in 60 seconds might produce an earthquake with a Richter magnitude of 7 to 8 or so. These are crude comparisons.

What about Richter magnitude, what is that? There are three common measures of event size, each of which is independent of the other (they have nothing in common, and cannot be compared directly). The oldest measure of size is the **Mercalli scale**, based upon the amount of damage and human response; this scale was

used before earthquake recording machines became sophisticated. The **Richter scale** is based upon the amount of ground shaking as measured on a recorder. The Richter scale is logarithmic; each Richter number represents a 10 fold increase in the amount of ground shaking. The standard surface wave formula is:

$$M_s = \log_{10}(A/T) + 1.66 \log_{10}(D) + 3.3$$

where A is the amplitude of the ground shaking in microns (a millionth of a meter), T is the period of the surface waves (in seconds) and D is the angular distance from the recorder to earthquake (in degrees). Note that this is just one standard form of the surface wave magnitude; there are others.

The **moment scale** is based upon the amount of fault movement and the size of the broken area on the fault. The moment scale is also logarithmic:

$$M_w = 2/3 \log_{10}(\mu Sd) - 10.7$$

where μ is the shear strength of the rock along the fault, S is the area of the fault and d is the displacement or slip along the fault. This scale is the one currently used by scientists in technical work. Scientists still calculate the Richter magnitude but mostly for consumption by the general public, which knows that a 4 is boring, but an 8 is a whopper.

You can see why the three scales are not comparable, and why the scientists use the moment scale for analyzing faults and earthquakes. Suppose you have a certain amount of fault movement. If houses are built poorly, you get a high Mercalli number; the opposite if construction is good. If the ground is soft and bouncy, you get a high Richter number; the opposite if it's strong and firm.

What **causes** the fault blocks to move suddenly and without warning? This is a crucial question, and a very contentious question. The traditional explanation is the "sticky hands" model. Lay your hands out flat, palms down, thumbs tucked in, and put your hands side by side so they are touching; these are the two fault blocks. The concept is that the fault is sticky, stress builds up, no movement takes place, stress builds and builds until finally the stickiness is overcome, and the hands move relative to each other. The sticky hands model is almost 100 years old, and it has its problems. One of the biggest problems is that the sticky hands model predicts that earthquakes are cyclic or periodic, that earthquakes will take place at regular intervals. The fault moves, releases stress, builds up, moves, releases stress, builds up, etc. in a cyclic or periodic fashion. But almost all faults are very non-cyclic; earthquakes don't appear in regular cycles. The sticky hands model also predicts earthquakes of approximately all one size, the size needed to overcome the stickiness. Earthquakes seem to be unpredictable in both time and size.

There are several non-traditional explanations of fault behavior, all based on some form of randomness and unpredictability. The earthquake exercise you will work on focuses on one of these theories, known as **self-organized criticality**. Faults are considered to have organized themselves into a critical state. The critical state of being is an unpredictable one; a small event may die out and have no great effect, whereas the next small event may amplify and grow and become enormous. Neither result is predictable.

An excellent example is given by the sandpile. Let sand grains pile up one grain at a time into a cone-shaped pile. A single sand grain falls on top of the pile; nothing happens. Another single sand grain falls on top of the pile, and a giant sand slide is created, sweeping away half the sandpile. The sandpile has organized itself into a critical state, where the end result of a very small trigger are

completely unpredictable. Another excellent example is auto accidents on the freeway. The cars speeding down the freeway are at a critical state. A driver swerves slightly and nothing happens. A driver swerves slightly, another driver jerks the wheel to get out of the way, bumps a car, which fishtails into another car, eventually resulting in a 17 car pileup. A small event may decrease to nothing, or may propagate into an enormous result. Neither result is predictable.

This concept of self-organized criticality has been applied to faults. The fault is at a critical state. A tiny amount of movement takes place along the fault. If the initial movement dies out, then a tiny earthquake is produced. If the initial movement propagates and grows and increases, a giant quake is produced. There is no predicting which will happen.

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6 Billion and Counting

Exercise # 10: Power law frequency distribution of quakes

Name _____ Name _____

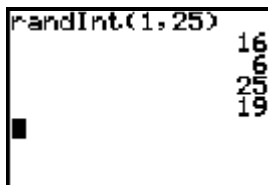
This exercise is designed to generate a population of artificial earthquakes, and to compare the model to real earthquakes. The earthquake generator is called a **cellular automaton**, but we prefer the name **bingo box**. The bingo box generates "bingoquakes."

Before Getting Started You will be using on your TI what computer scientists call a random number generator. Before using the random number generator for the first time, it is necessary to "seed" the calculator with an initial number (otherwise everybody's random numbers will be the same!) To seed your TI, make up a 9 digit number. What is your number ? _____ On the homescreen, type in your number, then press **[STO]**, then select RAND (found in MATH-PRB), finally press **[ENTER]**. Your number will be re-displayed and you are all set to proceed!

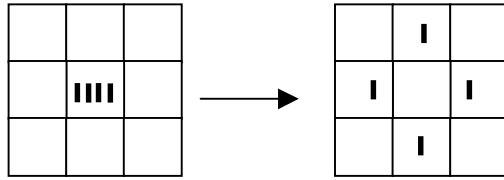
your number → rand
your number

Directions for the Bingo Box

Start with a 5x5 grid of squares, each square is labeled 1 through 25 in the upper left hand corner (provided for you). This is the bingo box. The rules of play are simple. Using the TI calculator, you will generate random numbers from 1 to 25. The formula to generate random integers from 1 to 25 is: **randInt(1, 25)**. The function **randInt** is found under **MATH - PRB**. Enter this on the Homescreen, then press ENTER repeatedly, see below.



Generate a random number. Mark that square with a **pencil** line (soft pencil the best). Repeat. Some squares will start filling up, others will remain blank. When a square reaches 4 tick marks, this is the start of a bingoquake. Erase the 4 tick marks, and put each of the 4 tick marks in the squares to the left, right, above and below. See below.



If a tick mark goes outside the bingo box, fine, it disappears. Your bingoquake is Size 1 **unless** an adjacent square has bumped up from a 3 to a 4 from one of your "jumping bean" tick marks. Erase those 4 tick marks and redistribute. Your bingoquake is now of Size 2 (2 squares were cleared out). Continue if necessary. **The total number of squares cleared out in one event is the size of your bingoquake.**

Record each bingoquake size in the table below the bingobox. The most efficient strategy is for one person to generate random numbers and the second person to run the bingo box, with the first person helping to keep track of where the jumping tick marks go and (most important) the eventual size of the bingoquake.

It's possible to end up with 5 or 6 tick marks in a square during a large bingoquake event. Just clear out 4 tick marks, keep the others in the square. It's also possible for a square to clear out and then fill up again during a large bingoquake; fine. Keep playing!

Run the bingo box for about a half hour or so. We will combine all of the bingoquake data together from the entire class to make a reasonable data set, hopefully with over 500 bingoquakes.

<< **WORKSHOP BREAK**>> As we compile the data, please talk to your neighbor, stretch, use the washroom, or take a look at the next two questions.

1. The Bingo box as a Critical System

One goal of the bingo box is to create a model system that showed **criticality**; a tiny "blip" would produce nothing, and the next identical blip would produce a giant event. Did the bingo box show critical behavior? Explain.

2. Is the Bingo Box Unpredictable?

A second goal of the bingo box model is to simulate the unpredictable nature of real earthquakes. What is it about this model that prevents us from predicting the size of the next quake?

**** **Record class data on size and frequency on the data sheet. Then enter size in L1 and frequency in L2. (Let us know if you need help!)**

3. The Bingo Box Frequency Distribution

Another goal was to generate size-frequency data with our model that look like size-frequency data for real earthquakes. Inspect the class data, tell us Yes or No whether the bingo box did the job, and explain briefly your answer. No math necessary, just eyeball the data.

4. Tweaking the Class Data: Cumulative Frequency

Before you plot or mathematically manipulate the data, you must reconfigure the frequency data. Enter, in the third column and in L3, the number of bingoquakes greater than or equal to that size. Let's start with Size = 1. How many bingoquakes are greater than or equal to 1 in size? Repeat for all bingoquakes. Call this new frequency the **cumulative frequency**. You probably noticed that the first cumulative frequency is N_T , the total number of bingoquakes. The second cumulative frequency is $N_T - N_1$, where N_1 is the number of bingoquakes of Size 1. The third cumulative frequency is $N_T - N_1 - N_2$, etc. etc.

Why are we converting to cumulative frequency, why not just use the actual frequency of each bingoquake? The cumulative frequency produces a nice decreasing sequence of numbers, and it smoothes out the frequency distribution, giving a pleasing result. If we use the plain frequency data, some bingoquake sizes may have 0 (zero) events of that size. The log of 0 is not defined, so that would cause problems for our calculator when it calculates the logs of all the data.

5. Power Law Regression

Using the class data and the PwrReg function on your TI, calculate the best fit power law regression between **size** and **cumulative frequency** for the bingoquakes. Note: On the Homescreen enter **PwrReg L1, L2**. PwrReg is found under STAT-CALC.

What is the best fit power law regression? What is the correlation coefficient?

How well do the data fit a power law relationship? Explain.

Use the power law equation to predict the cumulative frequency of a bingoquake of size 15 or greater.

Does your equation's prediction match the actual class data? Discuss.

Let's assume that all of the bingoquakes occurred in exactly 30 minutes. What is the temporal frequency of bingoquakes of size 5 and higher? (Note: temporal frequency is measured in bingoquakes per minute.)

The recurrence interval is the reciprocal of temporal frequency. Use your previous answer to find the recurrence interval of bingoquakes 5 and higher. Include units in your answer.

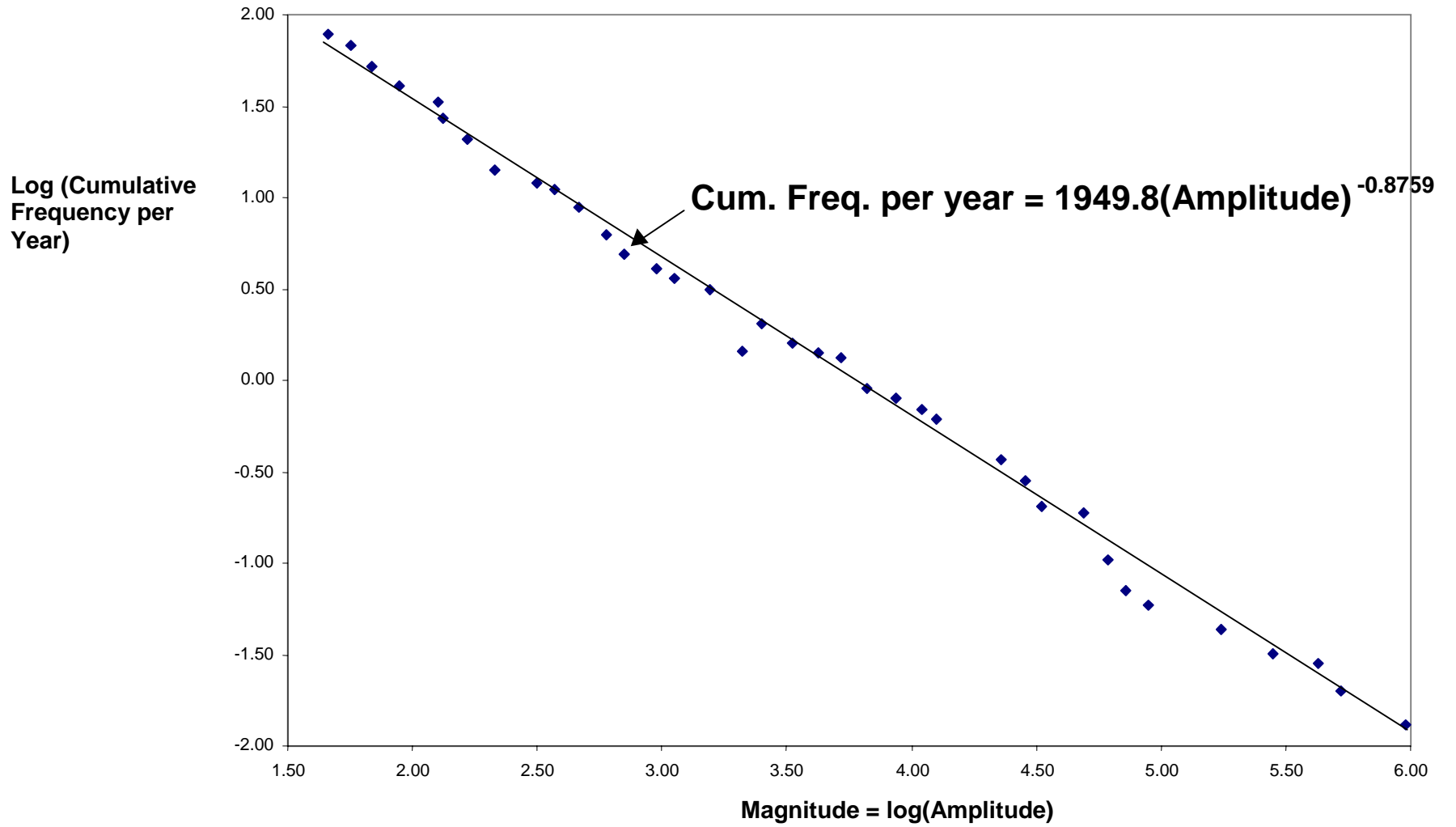
The 5x5 bingobox is theoretically capable of generating a bingoquake of size 36. Find the recurrence interval for a bingoquake of size 36. Interpret the meaning of your answer.

6. Comparison with Real Earthquake Data (New Madrid Fault in Missouri)

Do the New Madrid earthquake data (see attached scatterplot) show a power law frequency distribution of sizes like the bingoquake data? Explain your answer briefly.

Real earthquakes show a constant exponent (in the power law) over many orders of magnitude of earthquake size; see New Madrid scatterplot. Do the bingoquakes have a constant exponent, or is the exponent different for small versus large bingoquakes? Explain your answer briefly.

New Madrid Earthquakes



The 5x5 Bingo Box

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Number of Bingoquakes of Size X:

1.	7.	13.	19.	25.	31.
2.	8.	14.	20	26.	32.
3.	9.	15.	21.	27.	33.
4.	10.	16.	22.	28.	34.
5.	11.	17.	23.	29.	35.
6.	12.	18.	24.	30.	36.

6 Billion and Counting

Exercise # 10: Power law frequency distribution of quakes

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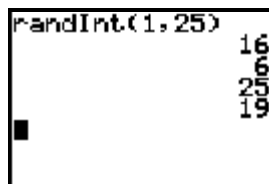
This exercise is designed to generate a population of artificial earthquakes, and to compare the model to real earthquakes. The earthquake generator is called a **cellular automaton**, but we prefer the name **bingo box**. The bingo box generates "bingoquakes."

Before Getting Started You will be using on your TI what computer scientists call a random number generator. Before using the random number generator for the first time, it is necessary to "seed" the calculator with an initial number (otherwise everybody's random numbers will be the same!) To seed your TI, make up a 9 digit number. What is your number? _____ On the homescreen, type in your number, then press **STO**, then select **RAND** (found in **MATH-PRB**), finally press **ENTER**. Your number will be re-displayed and you are all set to proceed!

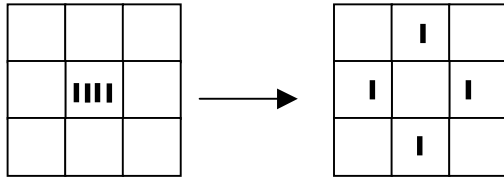
your number → rand
your number

Directions for the Bingo Box

Start with a 5x5 grid of squares, each square is labeled 1 through 25 in the upper left hand corner (provided for you). This is the bingo box. The rules of play are simple. Using the TI calculator, you will generate random numbers from 1 to 25. The formula to generate random integers from 1 to 25 is: **randInt(1, 25)**. The function **randInt** is found under **MATH - PRB**. Enter this on the Homescreen, then press **ENTER** repeatedly, see below.



Generate a random number. Mark that square with a **pencil** line (soft pencil the best). Repeat. Some squares will start filling up, others will remain blank. When a square reaches 4 tick marks, this is the start of a bingoquake. Erase the 4 tick marks, and put each of the 4 tick marks in the squares to the left, right, above and below. See below.



If a tick mark goes outside the bingo box, fine, it disappears. Your bingoquake is Size 1 **unless** an adjacent square has bumped up from a 3 to a 4 from one of your "jumping bean" tick marks. Erase those 4 tick marks and redistribute. Your bingoquake is now of Size 2 (2 squares were cleared out). Continue if necessary. **The total number of squares cleared out in one event is the size of your bingoquake.**

Record each bingoquake size in the table below the bingobox. The most efficient strategy is for one person to generate random numbers and the second person to run the bingo box, with the first person helping to keep track of where the jumping tick marks go and (most important) the eventual size of the bingoquake.

It's possible to end up with 5 or 6 tick marks in a square during a large bingoquake event. Just clear out 4 tick marks, keep the others in the square. It's also possible for a square to clear out and then fill up again during a large bingoquake; fine. Keep playing!

Run the bingo box for about an hour or so. You should end up with up to 100 bingoquakes of different sizes. We will combine all of the bingoquake data together from the entire class to make a reasonable data set, hopefully with over 1000 bingoquakes.

1. The Bingo box as a Critical System

One goal of the bingo box is to create a model system that showed **criticality**; a tiny "blip" would produce nothing, and the next identical blip would produce a giant event. Did the bingo box show critical behavior? Explain.

2. Is the Bingo Box Unpredictable?

A second goal of the bingo box model is to simulate the unpredictable nature of real earthquakes. What is it about this model that prevents us from predicting the size of the next quake?

**** **Now record your earthquake data on the front board.** We will sum all bingoquakes from each group on the board according to quake size. **Record class data on size and frequency on the data sheet. Finally enter size in L1 and frequency in L2.**

3. The Bingo Box Frequency Distribution

Another goal was to generate size-frequency data with our model that look like size-frequency data for real earthquakes. Inspect the class data, tell us Yes or No whether the bingo box did the job, and explain briefly your answer. No math necessary, just eyeball the data.

4. Tweaking the Class Data: Cumulative Frequency

Before you plot or mathematically manipulate the data, you must reconfigure the frequency data. Enter, in the third column and in L3, the number of bingoquakes greater than or equal to that size. Let's start with Size = 1. How many bingoquakes are greater than or equal to 1 in size? Repeat for all bingoquakes. Call this new frequency the **cumulative frequency**. You probably noticed that the first cumulative frequency is N_T , the total number of bingoquakes. The second cumulative frequency is $N_T - N_1$, where N_1 is the number of bingoquakes of Size 1. The third cumulative frequency is $N_T - N_1 - N_2$, etc. etc.

Why are we converting to cumulative frequency, why not just use the actual frequency of each bingoquake? The cumulative frequency produces a nice decreasing sequence of numbers, and it smoothes out the frequency distribution, giving a pleasing result. If we use the plain frequency data, some bingoquake sizes may have 0 (zero) events of that size. The log of 0 is not defined, so that would cause problems for our calculator when it calculates the logs of all the data.

5. Power Law Regression

Using the class data and the PwrReg function on your TI, calculate the best fit power law regression between **size** and **cumulative frequency** for the bingoquakes. Use $x = L1 = \text{size}$ and $y = L2 = \text{cumulative frequency}$. What is the best fit power law regression? What is the correlation coefficient?

How well do the data fit a power law relationship? Explain.

Use the power law equation to predict the cumulative frequency of a bingoquake of size 15 or greater.

Does your equation's prediction match the actual class data? Discuss.

Let's assume that all of the bingoquakes occurred in exactly 30 minutes. What is the temporal frequency of bingoquakes of size 5 and higher? (Note: temporal frequency is measured in bingoquakes per minute. Temporal means "time".)

The recurrence interval is the reciprocal of temporal frequency. Use your previous answer to find the recurrence interval of bingoquakes 5 and higher. Include units in your answer.

The 5x5 bingo box is theoretically capable of generating a bingoquake of size 36. (This is the maximum bingoquake possible for a 5x5 bingo box.) Find the recurrence interval for a bingoquake of size 36. Interpret the meaning of your answer.

6. Comparison with Real Earthquake Data (New Madrid Fault in Missouri)

Do the New Madrid earthquake data (see attached scatterplot) show a power law frequency distribution of sizes like the bingoquake data? Explain your answer briefly.

Real earthquakes show a constant exponent (in the power law) over many orders of magnitude of earthquake size; see New Madrid scatterplot. Do the bingoquakes have a constant exponent, or is the exponent different for small versus large bingoquakes? Explain your answer briefly.

7. What is earthquake magnitude?

The size of an earthquake can be measured by the amplitude (height) of the ground shaking; the more the ground is shaking up/down or side to side, the bigger the earthquake. Sizes of earthquakes vary over 10 orders, making it difficult to show earthquakes on one graph or compare earthquakes. So we take the logarithm of the amplitude, and call that "Richter magnitude."

- Richter magnitude = $\log(\text{amplitude of ground shaking})$
- Amplitude of ground shaking = $10^{\text{Richter magnitude}}$

What is the amplitude of an earthquake with Richter magnitude = 5.3?

What is the Richter magnitude of an earthquake with amplitude = 2500?

Can you have Richter magnitudes < 0 (negative Richter magnitudes)?

8. Cumulative Frequency with Time

The catalog of real earthquakes usually represents a certain time period, for example, the last 50 years, or from 1965-1995 (30 years). Therefore, the cumulative frequency is often normalized to the time period, to give **earthquakes per year** of a given magnitude or greater. (I.e., the temporal frequency.) The attached diagram shows earthquakes for the New Madrid seismic zone around Missouri from 1816-1983. For example, in this 167 year period, there have been (on average) about 1 earthquake per year greater than or equal to Richter magnitude 3.8.

What is the temporal frequency of New Madrid earthquakes greater than or equal to Richter magnitude 6? Express as: 1 quake per x years.

Calculate the temporal frequency of New Madrid earthquakes greater than or equal to Richter magnitude 8. Show work.

Richter magnitude 8 earthquakes took place in the New Madrid area in the winter of 1811-1812. IF the next magnitude 8 earthquake takes place with the average temporal frequency for earthquakes of this size or greater, when will that next magnitude 8 earthquake occur?

Can the cumulative frequency be used to predict earthquakes? Yes or No, and explain your answer. [A prediction must say when the next earthquake will take place (within uncertainty), and how big the next earthquake will be (within uncertainty).]

9. Size of the bingo box/size of the fault

Earthquakes are produced by fast movement along faults (breaks in rock). Faults are planar features that extend down into the ground. Because faults are planes, fault size is given by the area of the fault. The bigger the fault, the bigger the **maximum potential earthquake**; small faults have small earthquake potential, whereas big faults have the potential of producing big quakes. A scatterplot is provided (titled Global Earthquakes) that shows the maximum possible earthquake *moment* as a function of the fault area. (Note: *moment* is another measure of size.) These are real data for real faults and their real earthquakes.

In order to further test how well the bingo box is working as a fault/earthquake model, we can change the size of the bingo box, and see whether the maximum possible bingoquake changes in size as well. To find the maximum possible bingoquake, we **load up every square with 3 tick marks** ("saturate" the whole box), and then drop a 4th tick mark into a square **at or near the center** of the bingo box.

Load up a 3 x 3 bingo box, and find the maximum possible bingoquake size = ____ :

Load up a 4 x 4 bingo box, and find the maximum possible bingoquake size = ____:

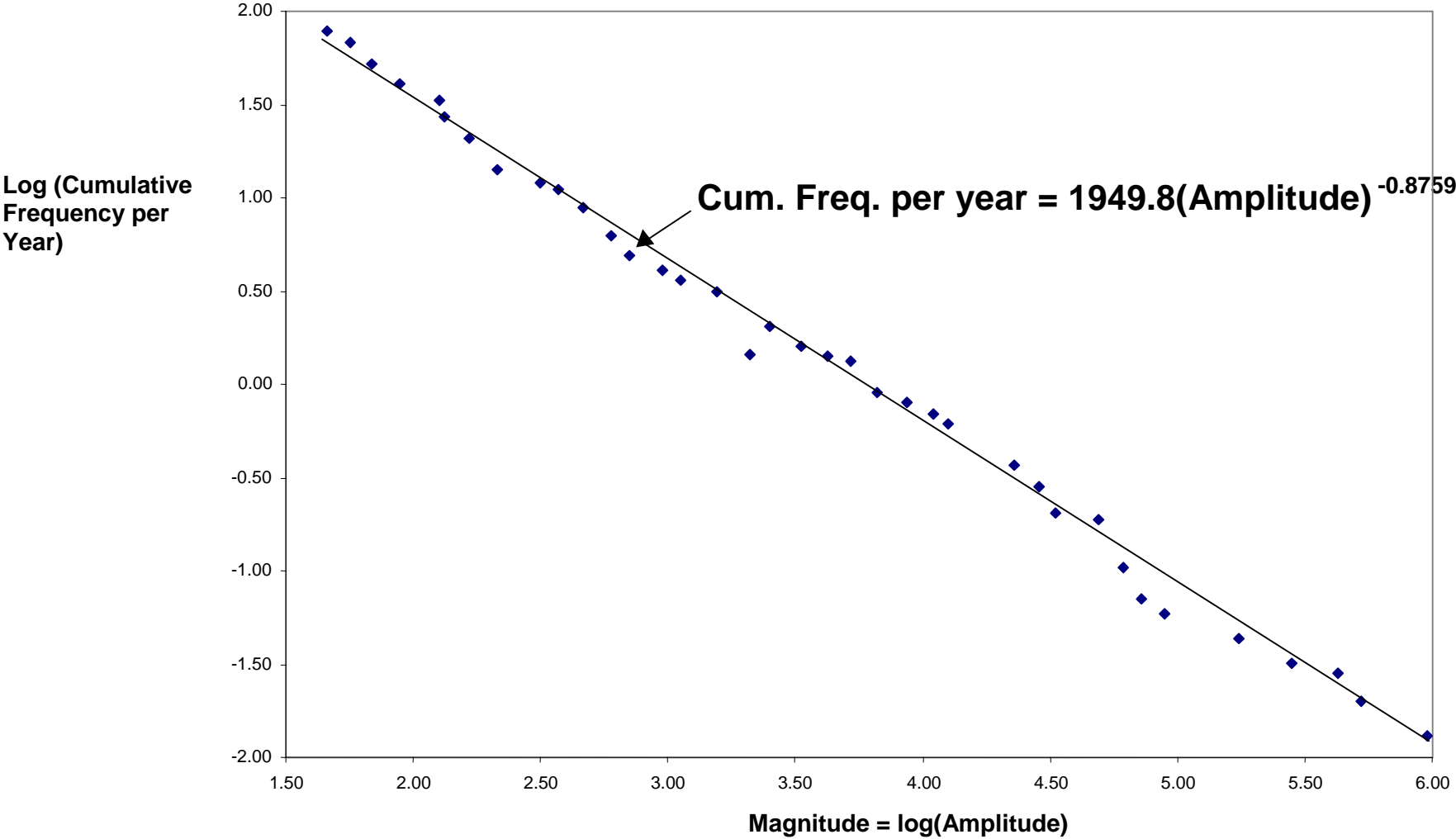
We've programmed this in Maple®, and let the computer do the work; study the table of bingo box area versus maximum size bingoquake (see attached paper). Did you get the same results as the Maple® program for a 3 x 3 and a 4 x 4?

Enter the area and maximum size data for the bingoboxes into your calculator. Try a few different curve-fitting functions for the computer-generated data. Which type of functional relationship gives the best fit to the computer-generated data? What is the equation of this function? What is the correlation coefficient?

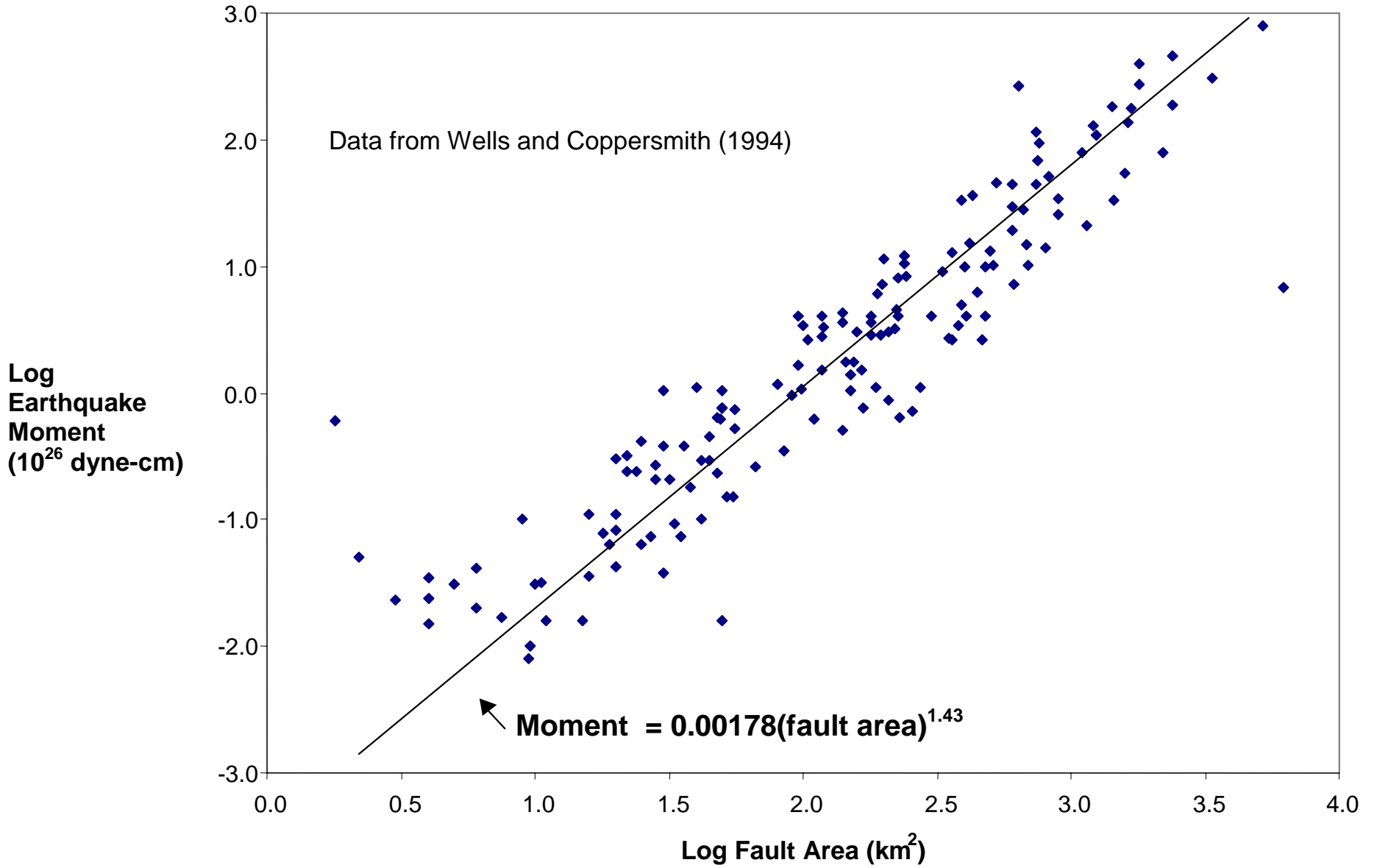
How does the area-maximum size relationship for the computer-generated data compare to the area-maximum moment relationship for real earthquakes? For example, do they show the same kind of functional relationship? If they do, how do the coefficients compare?

On this last point, how is the bingo box doing as a model for earthquakes?

New Madrid Earthquakes



Global Earthquakes



The following data were generated using Maple software.

Bingo Box Dimensions n x n	Area of Bingo Box	Maximum Size Bingoquake
n = 2	4	4
n = 3	9	10
n = 4	16	20
n = 5	25	35
n = 6	36	56
n = 7	49	84
n = 8	64	120
n = 9	81	165
n = 10	100	220
n = 11	121	286
n = 12	144	364
n = 13	169	455
n = 14	196	560
n = 15	225	680
n = 16	256	816
n = 17	289	969
n = 18	324	1140
n = 19	361	1330
n = 20	400	1540
n = 21	441	1771
n = 22	484	2024
n = 23	529	2300
n = 24	576	2600
n = 25	625	2925
n = 26	676	3276
n = 27	729	3654
n = 28	784	4060
n = 29	841	4495
n = 30	900	4960

The 5x5 Bingo Box

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Number of Bingoquakes of Size X:

1.	7.	13.	19.	25.	31.
2.	8.	14.	20.	26.	32.
3.	9.	15.	21.	27.	33.
4.	10.	16.	22.	28.	34.
5.	11.	17.	23.	29.	35.
6.	12.	18.	24.	30.	36.

Quantitative Environmental Learning Project (QELP)

As we prepared to teach our integrated course the first time, we were struck by the lack of interdisciplinary materials that were "class-room ready" and appropriate for our students. We would spend long afternoons searching the web for environmental data that could serve as a backdrop for the following day's student activity. If this sounds familiar, you know the quandary we were in! Slowly we built a collection of data sets and activities that (barely) served our purpose. We speculated that other faculty interested in teaching similar courses would appreciate accessing the small collection of data sets and materials we had assembled. In July, 2000 we established the Quantitative Environmental Learning Project (abbreviated as "QELP" and pronounced "kelp") with funding by the Division of Undergraduate Education at the NSF. QELP is a 2-year project that will create educational materials and host a workshop to encourage the establishment of interdisciplinary courses in mathematics and environmental science. Here are the details:

1. QELP Classroom Exercises

We will develop a source book with an array of exercises and solution sets, strategies for successful integration, guides to analyzing and interpreting the data and results, lessons learned, support materials and ancillary resources. The preliminary list of exercises is on page 7 of this handout. Please contact us if you are interested in working with (i.e. site testing) these exercises.

2. QELP Website Hosting Environmental Data Sets

We have established a website (www.seattlecentral.org/qelp) that, to date, hosts about 75 real data sets on numerous environmental science topics. These data sets are easy to download and are supported with background information. Feedback from educators suggests that these data sets can be used in a variety of ways, from simple to elaborate, to promote interdisciplinary learning. Graphs of environmental data can be displayed and discussed in lecture, or the actual data can be plotted and analyzed by students working in small groups. Students can access data sets and background information directly on the web for assignments or projects. Below is a snapshot of our data matrix indicating the number of data sets available in each category.

The screenshot shows the QELP website interface. At the top, there is a navigation bar with buttons for Home, Data, Links, New Book, Contact Us, and About. Below this is a secondary navigation bar with links for Data Home, Math Topics, Environment Topics, Topics Matrix (highlighted), Master List, Help, and Feedback. The main content area features a table with the following structure:

Frequent Users: Please Click Here!	BIVARIATE						UNIVARIATE	
	Linear	Exp	Logistic	Power	Periodic	Other	Histogram	Other
Ecology	1	2	1	1	0	0	4	0
Populations	0	0	3	0	1	0	3	0
Agriculture	1	1	0	1	0	0	1	0
Forests	0	0	0	0	0	2	1	0
Water Resources	3	1	0	1	0	3	2	0
Energy	2	1	1	2	2	1	0	0
Solid Waste	1	0	0	0	0	0	3	0
Water Pollution	4	1	0	1	0	1	2	1
Air Pollution	1	1	0	0	2	1	1	1
Hazards	0	0	0	1	0	1	2	1
Planet Earth	2	2	0	1	0	0	1	0

The browser window title is "QELP Data Page - Netscape". The address bar shows the URL: http://seattlecentral.org/qelp/Data_EnvironmentTopics.htm#Ecology. The taskbar at the bottom shows the Start button and several open applications, including HULL - N..., QELP ..., Exploring ..., Microsoft ..., and a folder named bof - Mic... The system clock shows 4:22 PM.

3. QELP Workshop

The Quantitative Environmental Learning Project will be hosting a 4 day workshop at Seattle Central Community College in Seattle, Washington from June 17-20 (Monday through Thursday), 2002. The workshop will focus on curricula that integrate mathematics and environmental science at the introductory college level. The workshop will be composed of two days of in-house sessions and two days of field trips (see tentative schedule below).

MONDAY 17 JUNE	TUESDAY 18 JUNE	WEDNESDAY 19 JUNE	THURSDAY 20 JUNE
Introduction, ice breaker ----- presentations sharing speakers	FIELD TRIP # 1 Puget Sound Beaches transects sampling censuses clam biometrics	FIELD TRIP # 3 Douglas Fir biometrics & forest ecology	Landslide Modelling: bingobox exercise ----- Create-An-Exercise
LUNCH	LUNCH	LUNCH	LUNCH
Urban Health Index ----- neighborhood measurements ----- results, discussion	preparation ----- FIELD TRIP # 2 Raging River discharge, velocity, transects, etc.	FIELD TRIP # 4 Mt. Rainier hazards lahars, mudflows, landslides eruptions	Exercise Results presentations ----- wrap-up, conclusion

A wide variety of in-house activities are planned, and plenty of time for free-form discussions and sharing will be included. We anticipate some formal presentations from the organizers, participants and guest speakers, focussing on strategies, materials and outcomes of interdisciplinary math and science education. We also anticipate a number of hands-on activities that you can take back to your classes, such as creating artificial landslides using a cellular automaton that models landslide recurrences, and quantifying an "urban health index" for a couple of city blocks adjacent to Seattle Central Community College.

One of the field trips will visit Mt. Rainier and vicinity. In the morning, participants will survey different aged forests along the White River corridor, from 300 year old virgin timber to third generation stick forests. Attendees will study the biometrics of Douglas firs and brainstorm about quantification of forest ecology. In the afternoon, we'll examine the geology of Volcano Rainier and learn about hazards, probabilities and recurrence intervals of both eruptions and debris flows.

A second field trip will visit both the Puget Sound shoreline and a Cascade foothills stream. In the morning, we will take advantage of a low tide to conduct biological censuses in the intertidal zone, and collect butter clam shells to take back to your classroom for biometric analysis. In the afternoon, we will measure stream transects across the Raging River and determine stream discharge from velocity and transect measurements, using state-of-the-art velocity meters and data collection units. We will also brainstorm about quantification of riparian ecology.

Much of the cost of this conference will be covered by the organizers, including housing at the Virginia Mason Inn on Capitol Hill near SCCC. A generous stipend will be provided to each attendee for participation.

We encourage pairs of instructors, as well as individuals, to apply. Priority will be given to pairs of instructors in mathematics and environmental/earth science from the same institution. Funding is available for up to 24 participants total.

If you are interested in attending this workshop, please fill out the application form on the next page and send it in.

Snail mail the workshop application form to:

Joe Hull
Division of Science and Mathematics
Seattle Central Community College
1701 Broadway
Seattle WA 98122
USA

Or you can fax the application form to:

Attn: Joe Hull
206-587-3837

If you want more information, contact Greg or Joe

Greg Langkamp, mathematics
(glangk@sccd.ctc.edu; 206-587-3810)

Joe Hull, environmental science
(jhull@sccd.ctc.edu; 206-587-4905)

Preliminary deadline for applications: 1 February 2002

**Application Form, QELP 2002 Workshop
Seattle, Washington
17-20 June 2002**

Name _____

Division or Department _____

School _____

Street/PO _____

City _____ State _____ Zip _____

Email _____

Phone _____ Fax _____

Teaching Discipline(s) _____

I plan to attend (check one) _____ by myself

_____ with another faculty whose name and discipline are:

(fellow faculty should send in their application too!)

On an additional sheet of paper, please describe:

4. your interest and experience in teaching interdisciplinary courses.
5. what you would like to see in the workshop, or get out of the workshop

Please remember that the deadline for this application is 1 February, 2002. Thanks!

Mail to:
Joe Hull
Division of Science and Mathematics
Seattle Central Community College
1701 Broadway, Seattle WA 98122

Fax to:
Attn: Joe Hull
206-587-3837