

Claim: The vertex of F can alternatively be given as $(h, k_0) \rightarrow (h, -ah^2)$.

Proof: Note that for F : ① The x intercepts are $(0, 0)$ and $(-\frac{b}{a}, 0)$

② the axis of symmetry is halfway between these points, $h = -\frac{b}{2a}$.

Since the vertex must lie on the axis of symmetry,

③ the x coordinate of the vertex is $h = -\frac{b}{2a}$ and the y coordinate is $k_0 = F(-\frac{b}{2a})$.

As shown below, the expression $F(-\frac{b}{2a})$ is equivalent to $-a(\frac{b}{2a})^2$:

$$\begin{aligned} F(x) &= ax^2 + bx && \text{original function} \\ F(-\frac{b}{2a}) &= a(-\frac{b}{2a})^2 + b(-\frac{b}{2a}) && \text{substitute } x = -\frac{b}{2a} \\ &= \frac{b^2}{4a} - \frac{b^2}{2a} \\ &= \frac{b^2 - 2b^2}{4a} \\ &= \frac{-b^2}{4a} && \text{simplify} \\ &= \frac{-b^2}{4a} \cdot \frac{a}{a} && \text{multiply by } \frac{a}{a} \\ &= -a\left(\frac{b}{2a}\right)^2 && \text{result} \end{aligned}$$

From $h = -\frac{b}{2a}$ we have $-h = \frac{b}{2a}$ and it follows that

$$\begin{aligned} F(-\frac{b}{2a}) &= -a(-h)^2 && \text{substitute } -h = \frac{b}{2a} \\ &= -ah^2 && (-h)^2 = h^2 \end{aligned}$$

This verifies $k_0 = -ah^2$. The vertex of F is $(h, k_0) \rightarrow (h, -ah^2)$ as claimed. ◆