

# **An Intermediate Algebra Course That's Serious About Applications**

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**Starting thoughts:**

What percent of students at your college will need in their future to:

- multiply polynomials
- add rational expressions
- rationalize denominators
- expand logarithmic expressions (such as:  $\log\left(\frac{xy^3}{\sqrt{z}}\right)$ )

**Discussion notes:**

At my college, we are teaching these skills to many students who won't need them at all. Is the situation similar at your school?

Many of our students are hoping to satisfy the college math requirement by eventually taking Introductory Statistics or Mathematical Ideas. It is for them that we created the new alternative course I will be describing to you, Applied Intermediate Algebra.

Students who need to take College Algebra later will continue to be taught these traditional topics (see previous page) in our current Intermediate Algebra.

Wouldn't the students who only need a second semester of algebra benefit more from topics such as:

- creating and reading graphs
- matching real-world relationships (data or graphs) with function or equation types
- the effect of each variable or constant in a function or other equation
- translating back and forth between questions/statements in English and mathematical procedures and findings

When I've spoken recently to teachers of non-math transfer courses (such as economics, biology, and physics), the complaints about student weaknesses are predominantly in these (see previous page) four areas.

We are trying to address these complaints or wish-lists by creating the new course Applied Intermediate Algebra. It will emphasize the four skills listed, at the expense of the traditional symbol manipulation.

I would like to tell you about our new course in some detail, using our Course Rationale, Catalog Description, and Topic Outline.

To be honest, I am hoping that there occurs a large-scale (i.e. nation-wide) reconsideration of which two-year college students need which skills. I expect an accelerating exchange of traditional symbol manipulation skills in favor of graph and function skills for the benefit of many.

Regarding a textbook for the new course, we are starting with the same one used by our Intermediate Algebra. It does have many data sets, graphs, and function models.

We also want it to be easy for students to switch early in the semester into and out of Applied Intermediate Algebra. The search for a book that's even more in line with the themes of our new course will continue.

## **COURSE RATIONALE FOR APPLIED INTERMEDIATE ALGEBRA**

Applied Intermediate Algebra is intended as a more appropriate prerequisite, than MCTC's current Intermediate Algebra, for two college-credit math courses, Introductory Statistics and Mathematical Ideas.

Most students who do plan to satisfy their college math requirements using the two courses mentioned above are not planning to take any more algebra. Therefore the symbolic manipulations of our current Intermediate Algebra would be of little benefit to them. In Applied Intermediate Algebra, this material is replaced by strong emphasis on applications and especially on current uses of algebra in various professions. The course thus becomes particularly interesting and beneficial.

The use of educational technology, and of graphing calculators in particular, is encouraged.

Every topic and every hour will be approached from an applications perspective!

There are a couple of points I would like to make here.

The first has to do with uses of algebra in various professions. We are planning to use actual data, charts/graphs, formulas, and functions from many fields. We expect this practice to increase students' hour by hour interest and to keep making the point that math is important to the work of a large variety of professionals.

Second, we want the instructor's ongoing concern to be: is the material I'm teaching today applicable? Am I planning to teach skills next hour that, realistically speaking, the student will never need in their eventual specialty? If so, skip them.

All of us had to learn traditional math operations... such as adding rational expressions or rationalizing denominators. It may be hard to drop them, but many of our students simply don't need them. They don't share our love of math, they don't appreciate the completeness or inter-relatedness that we see in the various chapters. I believe they will appreciate the usefulness of math if we just strive to make them aware of it.

## **CATALOG DESCRIPTION OF APPLIED INTERMEDIATE ALGEBRA** (3 Semester Credits)

Strong emphasis will be placed on applications of algebra and especially on current uses of function models (linear, quadratic, polynomial, rational, radical, logarithmic, and exponential). The relationship between data, graphs, and functions will be thoroughly explored. Translating statements and questions from English into mathematics, answering those questions using algebraic procedures, and then stating the results again in English will be discussed in depth. Three additional topics are: working with formulas, understanding variations, and solving equations that arise in applications. Only those symbolic manipulations that are related to the above activities will be studied. Prerequisite: Introductory Algebra (first semester of algebra, 5 semester credits). Applied Intermediate Algebra is the minimum algebra preparation for Introductory Statistics and Mathematical Ideas. Applied Intermediate Algebra does not prepare students (i.e. is NOT the prerequisite) for College Algebra.\*

\* We have our old course, Intermediate Algebra (2<sup>nd</sup> semester of algebra, 5 semester credits), that is the prerequisite for College Algebra (3<sup>rd</sup> and last semester of algebra, 4 semester credits).

Let me elaborate on one item here. We are teaching the students in this course the solution of only those equation types that arise in applications. Most often, this will mean answering questions related to a formula, a variation equation, a function, or a pair of functions.

## TOPIC OUTLINE

### TOPICS

Background: Applied Intermediate Algebra follows a 5-semester credit Introductory Algebra course. Also, at 3 credits, not much time is available. Be careful about the time spent reviewing.

### ADDITIONAL NOTES

All classroom examples should have an application context.

### APPLICATIONS OF LINEAR EQUATIONS IN ONE VARIABLE

Includes: defining your variable, attention to units, translating from English. Emphasis on realistic situations. Some of these will involve percents.

### INTRODUCTORY CONCEPTS IN MODELING

- Introduction to tables
- Pie charts and bar graphs
- Line and curve graphs
- Setting up graphs and plotting data
- Introduction to mathematical models

Answering questions using one and two variable tables,

Look at various types

Includes: Assigning variables to axes and choosing scales

Who creates models and who uses them? General introduction to fitting collected data with a line or curve

Skip topics such as:  
number sets, set notation, inequalities, absolute value, properties of real numbers, operations with real numbers, solving equations that do not represent applications, rules of exponents, scientific notation

### INTRODUCTION TO FUNCTIONS

- Definition of functions from an applications point of view

Use examples from a wide variety of fields, including linear, quadratic polynomial, radical, and rational types.

- Explanatory and response variables, and cause/effect
- Domain and range from an applications point of view
- Ways of representing functions (verbal, table, graph, equation)
- Function notation
- Graphing a function
- Evaluating or solving, to find output and input values
- Prediction and its limitations

The implied domain  
Positive vs. negative numbers, discrete vs. continuous

Also, using graphs to answer questions about output and input values

## LINEAR FUNCTIONS

Background: In the prerequisite course, students had already done quite a lot with one linear equation in two variables. Applied Intermediate Algebra must concentrate on a functions and an applications approach.

- Slopes and rates of change
- What is linearity in terms of change?
- Intercepts
- Horizontal lines and constant functions
- Graphing linear functions
- Obtaining linear functions from described relationships
- Obtaining linear functions from data

Interpretation of x intercept  
Interpretation of y intercept as "starting" value

Taking advantage of slope-intercept format

Brief introduction to least squares regression idea

## SYSTEMS OF LINEAR EQUATIONS (TWO EQUATIONS AND TWO VARIABLES)

- Defining and choosing units for the variables
- What are the implications of using two variables instead of one, of having two equations instead of one
- Translating applications into systems of equations or pairs of functions
- Graphing a pair of equations or functions on the same set of axes and answering questions with the help of the graphs
- Effects of differing intercepts and/or different rates (parallel lines, intersecting lines)
- Significance of the intersection point
- Solving systems using substitution and addition
- Systems that are redundant (dependent) or contradictory (inconsistent).

Too little information, the right amount of information, too much information

Background: There are numerous applications that have been traditionally been taught at this point. Due to time constraints, questionable applicability, and students' lack of experiences and intuition in these areas (motion and chemical mixtures, for example), only a few types will be covered in this course.

- Mixtures involving different dollar values
- Mixtures of investments
- Break-even analysis and supply/demand
- Competing revenue or competing cost functions

## **QUADRATIC FUNCTIONS**

Background: In the prerequisite course, students had already learned to solve quadratic equations. Concentrate here on applications and functions.

- Review: solving quadratic equations
- Story problems that translate into quadratic equations
- Quadratic function models
- Graphing quadratic functions
- Non-linear types of change (non-constant rate of growth)
- Symmetry in quadratic models and graphs

## **POLYNOMIAL FUNCTIONS**

- Background: In Introductory Algebra, students were already introduced to terminology, operations, and factoring. Concentrate on polynomial models.
- Examples of polynomial models and their graphs
- Evaluating polynomial functions and the problem of finding  $x$  values given  $y$

- Characteristics of polynomial graphs (smoothness, continuity, hills and valleys, maxima and minima)
- End behavior and its implications
- Relationships between degree and growth or complexity

## RATIONAL FUNCTIONS

- Domain, in applications
- Horizontal asymptotes in applications (for example, long-term stability)
- Vertical asymptotes in applications (for example, skyrocketing costs of 100% census completion)
- Rational equations. Solving equations of the form  $a = (b/x) + c$ , a necessary skill for working with rational functions and inverse variation.

Skip: Operations with rational expressions, complex rational expressions, traditional story problems such as work or travel.

## FORMULAS

- Effect of each variable
- Using the formula
- Units of measurement
- Solving for variables
- 
- Relationship to functions

## VARIATION

- Translating variation relationships from English into equations
- Purpose of variation constants and how to find them
- Using the equations
- Units of measurement
- The effect of each variable
- Relationships to functions

## RADICALS

- Review definition of square and cube roots
- Estimation and calculator evaluation
- Domain and range, in applications
- $\sqrt{\quad}$  and  $\sqrt[3]{\quad}$  types of growth, with comparison to  $x^2$  and  $x^3$
- Models involving radicals
- Solving radical equations of the form  $a = b\sqrt{x} + c$ , a necessary skill for working with radical functions

Skip: rational exponents, simplifying radicals, operations with radicals, rationalizing denominators, complex numbers

## EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- Introduction to exponential growth models
- Exponential vs. polynomial growth and decline
- Graphing exponential growth functions
- Interpreting the y-intercept
- Compound interest
- Uninhibited growth and the role of the growth constant
- Exponential decay and the role of the decay constant
- Definition of logarithms
- Logarithmic scales and their scientific uses
- Evaluating or solving exponential functions to find input and output values (i.e. solving  $a = b \exp(cx)$ )

## LEARNER OUTCOMES/OBJECTIVES

Upon satisfactory completion of this course, a student will:

- a. Understand the relationships between data, graphs, and functions, and will be able to answer a large variety of questions using a given table, graph, or function

And will be able to:

- b. Suggest a reasonable function type for a given graph
- c. Suggest a reasonable type of graph for a given function
- d. Translate some situations into equations or functions
- e. Determine the practical domains, and sometimes ranges, of given function models
- f. Solve linear, quadratic, polynomial, rational, logarithmic, and exponential equations, or linear systems, to the extent required by the given applications
- g. Determine whether the available data is sufficient or self-contradictory for the solution of some problems
- h. Use the predictive properties of models while being aware of the dangers of extrapolation or of assuming causation
- i. Use his/her calculator to evaluate polynomial, rational, radical, logarithmic, and exponential expressions