

Sample of College-Level Math Attributes in Statistics

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Typical Course Description:

This course introduces the students to the study of measures of central tendency, measures of variation, graphical representation of data, least squares regression, correlation, probability, probability distributions, sampling techniques, parameter estimation and hypothesis testing. The use of technology is integrated throughout the course. The emphasis is on applications from a variety of sources including newspapers, periodicals, journals and many of the disciplines that students may encounter in their college education. Students shall be expected to gather and analyze data, and formally report the results of their research.

Typical Outcomes:

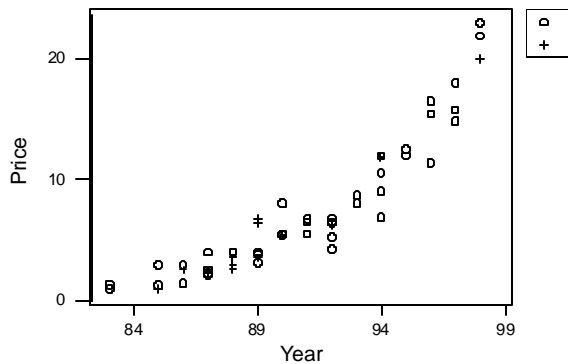
Students successfully completing this course should be able to:

- A. Understand the terminology used in statistics.
 - B. Understand the formulas used in statistics and be able to perform calculations using them.
 - C. Summarize data both graphically and in tables.
 - D. Fit least squares regression to data and understand the meaning of the terminology, measures and calculations used in regression.
 - E. Perform elementary probability calculations, and solve problems using standard probability distributions (e.g., discrete, binomial, uniform and normal).
 - F. Understand the sampling distribution of the mean and the central limit theorem.
 - G. Solve problems involving confidence intervals and parameter estimation.
 - H. Solve problems involving single sample hypothesis tests.
- Several of the following may also be required:
- I. Solve problems involving two-sample hypothesis tests and confidence intervals.
 - J. Solve problems involving one-way ANOVA.
 - K. Solve problems involving nonparametric tests.

Attributes	Sample Problems from College-Level Statistics	Prerequisite Problems or Skills from Intermediate Algebra
1, 2, 3, 4, 5, & 6.	<p>1. An economist wishes to estimate a line, which relates personal consumption expenditures (C) and disposable income (I). Both C and I are in thousands of dollars. She interviews 8 heads of households for families of size four and obtains the following data:</p> <p style="text-align: center;">C 16 18 13 21 27 26 36 39 I 20 20 18 27 36 37 45 50</p> <p>Let I represent the independent variable.</p> <ol style="list-style-type: none"> a. Use a graphing utility to draw a scatter plot. b. Use a graphing utility to fit a straight line to the data. c. Interpret the slope. The slope of this line is called the marginal propensity to Income. d. Predict the consumption of a family whose disposable income is \$42,000. 	<p>1. Each Sunday, a newspaper agency sells x copies of a certain newspaper for \$1.00 per copy. The cost to the agency of each newspaper is \$0.50. The agency pays a fixed cost for storage, delivery, and so on of \$100 per Sunday.</p> <ol style="list-style-type: none"> a. Write the equation that relates the profit P, in dollars, to the number x of copies sold. b. Graph your equation. c. What is the profit to the company, if 5000 copies are sold? d. How many copies must the company sell to make a profit of \$5000? e. Find the slope of the equation and give a one or two sentence narrative interpreting it within the context of this problem.

	<p>e. Predict the disposable income of a family whose consumption is \$30,000.</p> <p>f. What is the level of measurement for these two variables?</p> <p>g. Explain whether or not a linear model is advisable for this data.</p> <p>h. How strong is the relationship between these two variables? Cite specific measures.</p> <p>i. What percent of the variation is explained by the regression equation?</p>	<p>Intermediate Algebra Skills Needed: Linear equations, solving linear equations for the dependent variable, graphing lines and points, slope, etc.</p>
<p>1, 2, 3, 4, 5, & 6.</p>	<p>2. The following data comes from an Arizona newspaper in August. The hour of the day and the time spent in the sun until a person's skin begins to redden is given in the table below and it is available on the computer file Burn Time.</p> <p style="padding-left: 40px;">Time of Day 9,10,11,12,13,14,15,16 Minutes to Redden 34,20,15,13,14,18,32,60</p> <p>We are trying to see if there is a relationship between Time of Day and Minutes to Redden.</p> <p>a. What is the explanatory variable and what is its level of measurement?</p> <p>b. What is the response variable and what is its level of measurement?</p> <p>c. Use a graphing utility to draw a scatter plot.</p> <p>d. Use a graphing utility to find the curve of best fit for this data.</p> <p>e. Draw a scatter plot of this data along with the curve of best fit.</p> <p>f. What is the mathematical name for the curve that best fits this data?</p> <p>g. Explain why the correlation coefficient should not be used with this data.</p> <p>h. Based on the equation of best fit, how much time can you spend in the sun at 2:30PM before your skin begins to redden?</p> <p>i. If you needed to spend 20 minutes in the sun, based on your equation of best fit, for what interval(s) during the day would could you safely spend in the sun before reddening?</p> <p>j. How strong is the relationship between these two variables? Cite specific measures.</p> <p>k. What percent of the variation is explained by the regression equation?</p>	<p>2. The height H, in feet, of a projectile with an initial velocity of 96 ft./sec launched from 120 ft. above ground level is given by the equation $H = -16t^2 + 96t + 120$, where $t =$ time in seconds. Sketch the graph of this function and find the following.</p> <p>a. How many seconds after the launch is the projectile 128 ft. above the ground?</p> <p>b. What is the projectile's maximum height and when does it reach that height?</p> <p>c. How many seconds after the launch does the projectile return to the ground?</p> <p>Intermediate Algebra Skills Needed: Parabolas, graphing non-linear functions, solving quadratic equations, etc.</p>
<p>1, 2, 3, 4, 5, & 6.</p>	<p>3 The data shown in the scatter plot below represents the prices of used Honda Accords collected from two separate newspapers in</p>	<p>3. A model for the number of people N in a community college who have heard a certain rumor is $N = P[1 - e^{-0.15d}]$, where P is the total</p>

1999. The data itself can be found on the computer file Honda-Spring99.



We are trying to see if there is a relationship between Price and Year.

- What is the explanatory variable and what is its level of measurement?
- What is the response variable and what is its level of measurement?
- Explain why linear, quadratic or cubic relationships are not the appropriate for this data.
- Use a graphing utility to find the curve of best fit for this data.
- Draw a scatter plot of this data along with the curve of best fit.
- What is the mathematical name for the curve that best fits this data?
- Explain why the correlation coefficient should not be used with this data.
- Based on the equation of best fit, what should you expect to pay for a 1984 Honda Accord?
- If you could only spend \$8,000 (not one penny more), based on your equation of best fit, what was the latest model Honda Accord you could expect to purchase?
- How strong is the relationship between these two variables? Cite specific measures.
- What percent of the variation is explained by the regression equation?
- Fit a linear, quadratic, cubic and exponential equation to this data. Summarize the following results in a table: mathematical model, regression equation, coefficient of determination, expected price

rumor is $N = P[1 - e^{-0.15d}]$, where P is the total population of the community college and d is the number of days that have elapsed since the rumor began. In a community of 1000 students, find the following:

- How many students will have heard the rumor after 3 days?
- How many days will have elapsed before 450 students have heard the rumor?

Intermediate Algebra Skills Needed:

Finding, graphing and determining exponential functions, using logarithms as an inverse to exponential functions, etc.

	<p>of a 1992 Honda Accord, expected price of a 1980 Honda Accord. Use the information from this table to explain why the exponential model is the recommended model, even though some of the other models have a higher value for R^2.</p>	
1, 2, 3, 4, 5, & 6.	<p>4. We wish to determine whether grade point averages (GPA) differ for boys and girls. It is assumed that the GPA is normally distributed with an identical variance for both sexes. Two independent samples of five students each yield the observations listed below. Using a 0.05 level of significance, test whether or not the mean GPA for boys is the same as the mean GPA for girls.</p> <p>GPA for boys: 2.7 2.9 2.5 3.2 2.8 GPA for girls: 3.4 2.6 2.6 3.0 3.3.</p> <p>Note: The student will have to use the formula</p> $t = \frac{(\bar{X}_1 - \bar{X}_2) - (m_1 - m_2)}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$	<p>4. Intermediate Algebra Skills Needed: Here it is hard to pinpoint a specific problem from Intermediate Algebra that compares directly with this problem. Clearly one needs to be familiar with non-trivial algebraic formulas and be able to use substitution to evaluate them. The student should recognize the reasonableness of the answer obtained. In addition, the problem given requires higher-level problem solving skills that those typically found in Intermediate Algebra.</p>
1, 2, 3, 4, & 5.	<p>5a. Consider the Uniform Distribution:</p> $f(x) = \frac{1}{6}, -2 \leq x \leq 4,$ <p>find its mean and standard deviation, and find the following probabilities: $P(-1 < X \leq 2)$, $P(X \geq 1.4)$ and $P(X < 0)$.</p> <p>5b. If 80% of all Labrador Retrievers are black, find the probability that in a random sample of 35 Labrador Retrievers the number of black ones is: 26, at least 26, at most 26, and less than 26.</p>	<p>5. Intermediate Algebra Skills Needed: Both of these problems require a familiarity with function notation. The first problem requires graphing a problem with a limited domain and finding areas of geometric figures. The second problem requires higher-level problem solving skills than Intermediate Algebra. The student must first recognize that this is a binomial probability problem. Knowledge of factorials and binomial coefficients is also required (usually taught in Statistics).</p>
1, & 2.	<p>6. For the following boxplots, histograms, stem-and-leaf diagrams and density functions (not shown here) describe their shape (symmetric, skewed right, or skewed left). For those that are symmetric, give their axis of symmetry. In addition, comment on the variability of the various distributions.</p>	<p>6. Intermediate Algebra Skills Needed: Knowledge of families of functions, such as, $y = f(x) + c$, $y = f(x + c)$, and $y = f(kx)$ are needed. As is the concept of symmetry.</p>
3.	<p>7. Solve the following inequality for μ:</p> $-z_{\alpha/2} < \frac{\bar{X} - m}{\frac{s}{\sqrt{n}}} < +z_{\alpha/2}.$	<p>7. Intermediate Algebra Skills Needed: Solving inequalities and literal equations is needed, along with manipulating rational expressions.</p>
1, 2, & 3.	<p>8. A bank has 244 customers with balances from \$0 to over \$15,000. Two of their customers are listed in the table below.</p> <p>Customer Balance Percentile Z-Score</p>	<p>8. Intermediate Algebra Skills Needed: Setting up and solving two equations and two unknowns.</p>

	<p>Jan \$1,150 24 -1,50 Mark \$8,000 55 0.45</p> <p>a. Which customer has a balance that is closest to the mean? b. To the nearest whole number, calculate the approximate number of customers with balance below \$8,000. c. To the nearest whole number, calculate the standard deviation for this data.</p>																																									
1, 2, 3, 4, 5, & 6.	<p>9. We often look at time series data to see the effect of a social change or new policy. Here are data on motor vehicle deaths in the United States. Because motor vehicle deaths will tend to rise as motorists drive more miles, we look instead at the rate of deaths, which is the number of deaths per 100 million miles driven.</p> <table border="1" data-bbox="285 758 889 940"> <thead> <tr> <th>Year</th> <th>Rate</th> <th>Year</th> <th>Rate</th> <th>Year</th> <th>Rate</th> <th>Year</th> <th>Rate</th> </tr> </thead> <tbody> <tr> <td>1960</td> <td>5.1</td> <td>1968</td> <td>5.2</td> <td>1976</td> <td>3.3</td> <td>1984</td> <td>2.6</td> </tr> <tr> <td>1962</td> <td>5.1</td> <td>1970</td> <td>4.7</td> <td>1978</td> <td>3.3</td> <td>1986</td> <td>2.5</td> </tr> <tr> <td>1964</td> <td>5.4</td> <td>1972</td> <td>4.3</td> <td>1980</td> <td>3.3</td> <td>1988</td> <td>2.4</td> </tr> <tr> <td>1966</td> <td>5.5</td> <td>1974</td> <td>3.5</td> <td>1982</td> <td>2.8</td> <td>1990</td> <td>2.2</td> </tr> </tbody> </table> <p>a. What is the level of measurement for Rate? b. What is the level of measurement for Year? c. Is this study observational or experimental? d. Draw a line-type time series plot (not a bar chart type) of this death rate data and describe the overall pattern of this data (how it varies over time). Use year as the independent variable and label your axes. e. In 1974 the national speed limit was lowered to 55 miles per hour in an attempt to conserve gasoline after the 1973 Mid-east war. In the mid-1980s most states raised speed limits on interstate highways to 65 miles per hour. Some said that the lower speed limit saved lives. Explain if the effects of the lower speed limits between 1974 and the mid-1980s are visible in your plot.</p>	Year	Rate	Year	Rate	Year	Rate	Year	Rate	1960	5.1	1968	5.2	1976	3.3	1984	2.6	1962	5.1	1970	4.7	1978	3.3	1986	2.5	1964	5.4	1972	4.3	1980	3.3	1988	2.4	1966	5.5	1974	3.5	1982	2.8	1990	2.2	<p>9. Intermediate Algebra Skills Needed: Choosing and labeling axes, plotting points, drawing lines and familiarity with a variety of graphs.</p>
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1, 2, 3, & 5.	<p>10. If $P(A) = 0.4$, $P(B) = 0.2$ and $P(A/B) = 0.5$, find the following: $P(A \cap B)$, $P(B/A)$, and $P(A \cup B)$. What must we change the value of $P(A)$ to, in order to make events A and B independent events?</p>	<p>10. Intermediate Algebra Skills Needed: Familiarity with function notation, solving literal equations, and substitution.</p>																																								
1, 2, 3, 4, 5, & 6.	<p>11. Jim and Yvette plan to have a family of three children, with girls being just as likely as</p>	<p>11. Intermediate Algebra Skills Needed: Familiarity with function notation, substitution,</p>																																								

	boys. Find the probability distribution for the number of girls and the probability that all of their children are boys.	and higher-order problem solving skills.
1, 3, & 5.	12. Find the mean, variance and standard deviation for the probability distribution given: $\begin{array}{cccccc} X & 1 & 2 & 3 & 4 \\ P(X) & .1 & .3 & .2 & .3 \end{array}$	12. Intermediate Algebra Skills Needed: Familiarity with function notation, and substitution in complex algebraic formulas.
1, 2, 3, 4, 5, & 6.	13. With increased airline traffic, it is predicted that in the next century airline accidents will average one a week. If they follow a Poisson distribution, what is the probability that the number of airline accidents in a month (4.3 weeks) is at least 7? In addition, find the mean, variance and standard deviation.	13. Intermediate Algebra Skills Needed: Familiarity with function notation, substitution in complex algebraic formulas, exponential functions and higher-level problem solving skills.
1, 2, 3, 4, 5, & 6.	14. A police officer buys a box of 13 jelly donuts (a baker's dozen). Nine of the donuts are strawberry and four are raspberry. If the police officer randomly selects and eats 5 of the donuts, what is the probability that he or she eats at least 2 strawberry jelly donuts? In addition, find the mean, variance and standard deviation.	14. Intermediate Algebra Skills Needed: Familiarity with function notation, substitution in complex algebraic formulas and higher-level problem solving skills.
1, 2, 3, 4, 5, & 6.	15. Suppose that of all students, who took a standardized math test, their average score was 980 with a standard deviation of 100. If test scores are normally distributed, a. Find the probability that a randomly selected student has a test score between 955 and 1000 inclusive. b. Find the probability that a random sample of 25 students had an average test score between 955 and 1000 inclusive.	15. Intermediate Algebra Skills Needed: Using formulas and tables, substitution, and higher-level problem solving skills.
1, 2, 3, 4, 5, & 6.	16. Janice wanted to estimate mean number of hours her dog spent sleeping each day. In sample of 29 days, she found that her dog slept for an average of 18.7 hours with a standard deviation of 4.7 hours. a. Find the 99% confidence interval for the true mean number of hours her dog sleeps each day. Assume that her dog's sleep time is normally distributed. b. How large of a sample would you need if you wanted to be 99% confidence with a margin of error of no more than 1 hour?	16. Intermediate Algebra Skills Needed: Familiarity with function notation, substitution in complex algebraic formulas, using and solving inequalities, using tables, and higher-level problem solving skills.
1, 2, 3, 4, 5, & 6.	17. The Associated Press reported that in a recent survey of 8000 women, 648 said that they were stalked at least once. Stalking was	17. Intermediate Algebra Skills Needed: Familiarity with function notation, substitution in complex algebraic formulas, using and

	<p><i>defined by researchers as: “a course of conduct directed at a specific person that involves repeated physical or visual proximity, nonconsensual communication, or verbal, written or implied threats.”</i></p> <p><i>a. Find the 99% confidence interval for the true proportion women who feel they were stalked.</i></p> <p><i>b. How large of a sample would you need if you wanted to be 99% confidence with a margin of error of no more than 3%?</i></p>	<p>solving inequalities, using tables, and higher-level problem solving skills.</p>
1, 2, 3, 4, 5, & 6.	<p>18. <i>In a recent survey, it was discovered that 14 out of 50 people owned subcompact cars. In a related survey, it was found that 18 out of 100 people owned luxury cars. Can we conclude at a 0.05 significance level that subcompact cars are more popular than luxury cars?</i></p>	<p>18. Intermediate Algebra Skills Needed: Using formulas and tables, substitution, and higher-level problem solving skills.</p>
1, 2, 3, & 5.	<p>19. <i>Suppose that we are testing $H_0: m = 45$ and it is appropriate to use the z-test. If the value of our statistical formula is $z = -1.23$, find the p-value that corresponds to the following possible alternative hypotheses: $H_1: m > 45$, $H_1: m < 45$ and $H_1: m \neq 45$.</i></p>	<p>19. Intermediate Algebra Skills Needed: Using formulas and tables, substitution, and higher-level problem solving skills.</p>
1, 2, 3, 4, 5, & 6.	<p>20. <i>A test is normally distributed with a mean of 64.3 and a standard deviation of 7.2. Find the highest B if the top 10% of the students receive an A.</i></p>	<p>20. Intermediate Algebra Skills Needed: Using formulas and tables, substitution, inverse functions, and higher-level problem solving skills.</p>