Sample of College-Level Math Attributes in a Liberal Arts Mathematics Course

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Introduction

Connections between College-level Math and Intermediate Algebra

Students who have only completed elementary algebra have had minimal exposure to mathematical concepts. The focus of the first course in algebra is primarily linear equations with a bit of quadratic equations. Much of the course is spent on developing skills in manipulating symbols.

In Intermediate Algebra, the manipulative skills are further developed but more importantly the student is introduced to a much larger repertoire of functions including quadratic functions, exponential functions and radical equations. It is this broader view that helps to prepare the student for the more abstract ideas that will be presented in college-level mathematics. Although it is frequently possible to present abstract ideas in a 'plug-n-chug' fashion, this does not develop abstract thinking skills and defeats the purpose of studying mathematics.

Course Description:

In courses of this type, students develop the ability to reason with quantitative information through the study of the principles of reasoning, number sense, probability and statistical reasoning, unit analysis and mathematical modeling. Students will acquire the specific background and critical thinking skills they need to understand the major issues they will face in life, both on a personal level and as citizens in a modern democracy. There is an emphasis upon the contemporary applications to various real-life problems. This course is not for students who intend to major in mathematics or sciences.

Typical Outcomes for a Liberal Arts Mathematics Course

Goal: To help students be successful in using mathematics in the rapidly changing work environment they will face in any career.

As a Core Group of Competencies. Students completing a Liberal Arts mathematics course will demonstrate the ability to solve problems in the following areas:

I. Counting and Probability
   a. Solve counting problems involving permutations and combinations
   b. Calculate simple and compound probabilities
   c. Apply the law of large numbers and expectations to practical situations

II. Introductory Statistics
   a. Calculate and interpret statistical measurements of central tendency and dispersion
   b. Explain the relationship between statistical inference and probabilities

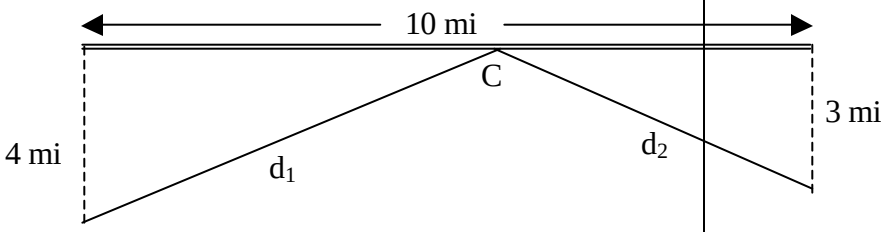
III. Exponential Growth
   a. Solve and interpret problems in finance, and/or

b. Solve and interpret problems involving population

IV. Network Analysis
   a. Apply network analysis to solve scheduling problems

In addition some of the following optional topics will also be required.

I. Logic
II. Linear Programming
III. Sets
IV. Voting Theory
V. Optimization
VI. Polygons and Polyhedra
VII. Game Theory

Attributes	Sample Problems from Liberals Arts Math	Prerequisite Problems or Skills from Intermediate Algebra
1,2,3,4,5,6	A team of employees with a cable company has been asked to determine routes for laying cable between satellite dishes and switching stations. They need to determine the placement of switching station C to service two new satellite dishes at points A and B. They want to minimize the amount of cable used. (from Applying Algebraic Thinking to Data by DeMarois, McGowan, & Whikkanack) 	Distance formula & Pythagorean Theorem Squaring binomials Radical Equations
1, 2, 3, 4, 6	If the course includes Voting Theory and Apportionment Methods, students need to be able to calculate election results based on various formulas and discern and discuss the advantages and disadvantages of the various methods for different situations. Sample formulas discussed in Voting Theory include: Sum of all Borda counts with *c* candidates and *v* voters = $vc(c + 1) / 2$; the threshold divisor for n seats under the Hill-Huntington method = population of the state divided by the square root of $(n-1)n$. Sample Problem: In the 1968 election a third party candidate actually carried a state. (George Wallace of the American	Ability to read and interpret mathematical information and formulas; ability to solve a 3 x 3 linear system.

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	Independent party won five southern states.) After dropping the one-third of one percent who voted for some other candidate, the actual vote is given in the first column of the table. A Gallup poll estimation of the breakdown by political party is shown in the final three columns. % all votes % Republican % Democrat % Indep. Nixon 43.5 86 12 44 Humphrey 42.9 9 74 31 Wallace 13.6 5 14 25 Although Nixon won in the Electoral College, the popular vote was very close. To analyze the election results, find the percentage of voters who are Republican, Democratic and Independent voters. Use the set of equations: 0.86R + 0.12D + 0.44I = 43.5 0.09R + 0.74D + 0.31I = 42.9 0.05R + 0.14D + 0.25I = 13.6 R + D + I = 100 Suppose that within each of the three classes of voters, the second choices are proportional to the percentage supporting the other two candidates. Create a table showing the percentage of voters having each possible preference ranking. Estimate who would have won the popular vote in a runoff between Nixon and Humphrey and estimate who would have won the popular vote in an election between Nixon and Wallace. (from *Mathematics Beyond the Numbers*, Gilbert & Hatcher)	
1,3,5,6	Sample formulas typically included in a Finance unit in a Liberal Arts Math course Compound Interest $A = P(1 + i)^n$ Annuities and Sinking Funds $A = \frac{R((1+i)^n - 1}{i}$	If solving for A, use of exponents If solving for P, negative exponents, understanding of inverse functions If solving for i, exponential functions If solving for n, understanding of inverse functions, logarithmic functions.

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		Sometimes explained using geometric series
	The concept of Inverse is introduced in Intermediate Algebra and then used in various ways in college-level classes. The following examples flow from simple to more elaborate and include several functions.	
2, 3, 4, 6		Example: Conversion between two monetary systems. US dollars to British Pounds 1.63 to 1 d = 1.63p and p = d/1.63
	1. Linear Salary function. y = 1500x + 35000 where y = year's salary and x = number of years of service. Inverse: $x = \frac{y - 33000}{2000}$	1. Ability to solve a linear function for a different variable
	2. Rational Function. 10 ounces of a mixed drink is 12% alcohol. Club soda is added to reduce the alcohol content. $C(x) = \frac{1.2}{10 + x}$ where C is the concentration of alcohol and x is the amount of club soda added. Inverse: $x = \frac{1.2}{C} - 10$ $x = \frac{1.2}{C} - 10$	2. Skills with rational equations; Prior exposure to percent mixture problems increases the student's ability to work with this type of problem.
	3. Quadratic. If an object is dropped from a height of 448 ft, the function is $d(t) = 448 - 16t^2$ Inverse: $t = \pm\sqrt{\frac{d}{16} - 28}$	3. Used in physical science, physics as well as math. Skills quadratic formula, radical equations
	4. Exponential. The average value of a house in 1990 was approximately \$93,000. Housing values in this area have increased at a rate of 6% every 3 years. Students should be able to model the function and solve for both the average housing value for a specific number of years (exponential equation) and solve for the number of years needed to reach an average housing value (logarithmic equation). $A(t) = 93{,}000\ (1.05)^t$	4. Exponential notation and problem solving is first introduced in intermediate algebra; logarithms

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	Many Liberal Arts Math courses include a unit on Introductory Probability & Statistics. Just like algebra, this area of mathematics is very broad and includes some topics that are taught in elementary school and others that require a grounding in calculus. A college-level treatment of topics in this area should require a student to think abstractly not just utilize formulas.	
1, 2, 3, 4, 5, 6	Bayes' Formula combines many of the topics covered in a probability unit. It is the inverse of a typical conditional probability. Using conditional probability – What is the probability that a patient was cured given that she was in the control group? Using Bayes'Theorem – What is the probability that a patient was in the control group given that the patient was cured? *Statement of Bayes' Theorms: Suppose $U_1, U_2, \ldots U_n$ are mutually exclusive events whose union is the entire sample space. Let E be an event with nonzero probability, Then* $\Pr(U_i \mid E) = \frac{\Pr(U_i \cap E)}{\Pr(U_1 \cap E) + \Pr(U_2 \cap E) + \ldots + \Pr(U_n \cap E)}$	The manipulative skills might be at the arithmetic level, but the concepts put it well beyond elementary algebra.
1, 2, 3, 4, 5, 6	The Binomial Formula relates the use of counting techniques to finding the coefficients needed when raising a binomial to a power greater than 1. $(x + y)^n = {}_nC_n\, x^n + {}_nC_{n-1}\, x^{n-1} y + {}_nC_{n-2}\, x^{n-2} y^2 + {}_nC_{n-3}\, x^{n-3} y^3 + \ldots + {}_nC_1\, x\, y^{n-1} + {}_nC_0\, y^n$	
	Several courses include new and emerging mathematics topics such as fractals or game theory.	
1,2,3,4,5,6	In Game Theory, students have to learn how to represent data graphically (using trees, table, and matrices) and analyze it critically.	Translate and organize information into tables, systems of linear inequalities.

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1, 2, 3, 4, 6	Fractals include objects that exhibit the quality of self-similarity (they contain repeated but smaller versions of the same larger pattern) and that have fractional dimension (for example, the self-similarity of the Koch Snowflake has the property that the object isn't exactly one-dimensional [like a straight line] nor is it two-dimensional [like a square]; rather, it is somewhere in between those dimensions). The self-similarity dimension of a fractal is determined using the formula: $d = \frac{\log N}{\log \frac{1}{r}}$ $d = \frac{\log N}{\log \frac{1}{r}}$, where N is the number of pieces the original piece has been divided into, and r is the reduction factor (the ratio of the length of the new piece to the length of the original piece). 1. The Koch Curve is constructed using the following procedure: a. Begin with a line segment, length L. b. Divide the segment into three equal parts. c. Remove the middle segment and replace it with an upside down V whose sides can be considered two sides of an equilateral triangle. d. Repeat this process on each of the four line segments to construct the next iteration. e. Repeat the process on each of the resulting line segments again to construct the third iteration. The Curve, itself, can contain an infinite number of iterations constructed as described above. However, answer the following questions based on the construction above: a. How many segments did you observe after constructing each iteration? b. If you were to construct the fourth iteration, how many segments would your curve contain? c. Determine the self-similarity dimension of the fractal curve at the third iteration. 2. Find three examples in our world that exhibit fractal properties. Explain why each can be considered a fractal.	The study of fractals includes the arithmetic concept of ratio; the geometric studies of dimension, measurement, length, and properties of shapes (the triangle was used in the example here); and the algebraic (intermediate level) concept of logarithmic functions.

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	The center of a circle is also the center of a square of side 2 cm. The circle passes through the four vertices of the square. Find the circumference of the circle. (Leave your answer in terms of π). The center of a circle is also the center of a square of side 2 cm. The circle passes through the four vertices of the square. Find the area of the region that is inside the circle and outside the square.	A rectangular plot of ground is to be enclosed with 120 yd of fencing. If the plot is to be twice as long as it is wide, what must be its dimensions? A rectangular reflecting pool in a park is 20 feet wide and 30 feet long. The park gardener wants to plant a strip of grass of uniform width around the edge of the pool. She has enough seed to cover 336 square feet. How wide will the strip be? A couple want to buy a rug for a room that is 20 feet long and 15 feet wide. They want to leave an even strip of flooring uncovered around the edges of the room. How wide a strip will they have if they buy a rug with an area of 234 square feet. A window is in the shape of a rectangle surmounted by a semicircle. The width of the window is 3 ft., which is also the diameter of the circle, and the height of the rectangular part is 4 ft. Find the total area of the window.