A Definition of College Intermediate Algebra by a Joint Committee of Two and Four Year College Mathematics Faculty for the Discussion of Mathematical Issues

What is Intermediate Algebra?

There cannot be an exact answer to this question, unless the curriculum at all of the colleges in Maryland is standardized. Since this is not a practical, nor a desirable objective, a less precise definition will have to suffice. First of all, to answer this question, it was felt that a comparison between Introductory Algebra and Intermediate Algebra is necessary. Secondly, it is recognized that Intermediate Algebra is ideally a level of mathematical maturity, that can be achieved though a variety of topics and skills. Finally, Intermediate Algebra is not static; it is changing and evolving along with the technology available to teach mathematics. Although we will not give a precise definition of Intermediate Algebra here, we hope that once you have perused this document, you will be able to recognize Intermediate Algebra when you see it.

It is felt that a comparison of Introductory Algebra and Intermediate Algebra is necessary, because some topics found in Intermediate Algebra at one college may be found in Introductory Algebra at another college. Intermediate Algebra comes at the end of the sequence of developmental math courses. It is this sequence of courses that is, in reality, the true prerequisite to "College Level" mathematics courses; therefore, it is not important in which developmental course individual topics appear. Intermediate Algebra should be thought of as a level of mathematical maturity. It is a combination of computational skills, manipulative skills and critical thinking skills needed to "think" mathematically. These skills can be achieved through a variety of topics and pedagogical techniques.

In general, one could say that, in Introductory Algebra, single skills or concepts are learned one at a time, with most manipulations requiring only a few steps. In Intermediate algebra, multiple skills or concepts are combined and most manipulations require many steps. Greater synthesis of the concepts is expected of students in Intermediate Algebra, and a higher level of critical thinking is required.

In this document, algebra is broken-up into two broad skill areas: Graphing & Function Notation and Solving Equations and Inequalities. In addition, a third category Manipulative Skills not Implied by the Previous Topics is included at the end. As its title suggests, this latter topic is a catch all for skills not implied in the other two broad categories. Each of these broad categories is further broken-up into smaller sub-categories or skill areas. In each of these smaller skill areas, a general description of what is "typically" covered in Introductory Algebra and Intermediate Algebra is given, followed by a couple of example problems. The example problems are usually ones that students generally find the most difficult. Essentially these "harder" problems are being used to indicate the depth to which the topic is covered and the level of mathematical maturity needed.

It is not the intention of this document to imply that every Introductory Algebra course or Intermediate Algebra course has to include all of these problems, or all of the sub-categories. These problems are only being used to indicate the level of mathematical maturity expected of students who have completed an Intermediate Algebra course. As mentioned earlier, some schools may not use all of the topics listed to teach students to achieve that mathematical maturity. There may be some schools that use topics that are not listed here, but it is felt that students should have mastered the preponderance of topics listed by the time they have completed their Intermediate Algebra course. For a variety of economic and political reasons, some schools do not introduce technology, specifically the graphing calculator, in their Developmental Math courses. Many of the examples given in this document are clearly written with the graphing calculator in mind, so colleges that do not use graphing calculators will probably not teach all of those skills. Once again, it is the level of mathematical maturity that is being illustrated here, and not the pedagogical approaches to teaching mathematics. Note: most of the problems given have been taken from typical textbooks used to teach Introductory Algebra and Intermediate Algebra.

There are topics omitted from this document, such as, Sequences, Mathematical Induction, Probability, Binomial Theorem, Sets, Permutations & Combinations, etc., that some colleges include in Intermediate Algebra. The list of topics is not meant to be an exhaustive list of every possible skill or topic. For a student to be on the level of Intermediate Algebra, he or she should have mastered a large majority of the skills listed here, and for those skills the student has not seen, he or she should mathematically mature enough to learn them with ease.

Graphing & Function Notation

	Introductory Algebra	Intermediate Algebra
Straight Lines	Coordinate axes are introduced, along with plotting of points and lines. Intercept and slope are introduced along with the slope intercept and point slope formulas. Numerically integers and easy fractions are used in nearly all of the problems. Applications concentrate on finding the equation of a line given either two points or the slope and one point, finding equations of parallel or perpendicular lines, and finding the point of intersection of two lines. The hardest application problems for students are typified by the following: *The points (-1, 1), (4, 1) and (4,-5) are three vertices of a rectangle. Find the coordinates of the fourth vertex.* *Find the equation of the line that contains the point (0,6) and is perpendicular to y = 3x –5.*	Topics from Introductory Algebra are reviewed and extended. Distance between to points and the midpoint of a line segment are introduced. Numerically some of the problems involve points whose coordinates are irrational numbers. Fitting lines to data and correlation are introduced. Application problems are more complicated than those of Introductory Algebra. The hardest application problems for students are typified by the following: *If (2, 0) and (0, 5) are points on the graph of y = mx + b, what are m and b?* *Each Sunday, a newspaper agency sells x copies of a certain newspaper for $1.00 per copy. The cost to the agency of each newspaper is $0.50. The agency pays a fixed cost for storage , delivery, and so on of $100 per Sunday.* *a. a. Write the equation that relates the profit P, in dollars, to the number x of copies sold.* *b. Graph your equation.* *c. What is the profit to the company, if 5000 copies are sold?* *An economist wishes to estimate a line, which relates personal consumption expenditures (C) and disposable income (I). Both C and I are in thousands of dollars. She interviews 8 heads of households for families of size four and obtains the following data:* *C| 16 18 13 21 27 26 36 39* *I| 20 20 18 27 36 37 45 50* *Let I represent the independent variable.* *a. Use a graphing utility to draw a scatter plot.* *b. Use a graphing utility to fit a straight line to the data.* *c. Interpret the slope. The slope of this line is called the* ***marginal propensity to Income.*** *d. Predict the consumption of a family whose disposable income is $42,000.*
Quadratic, Polynomial, & Rational Equations	The mechanics of graphing quadratic equations are stressed. Students are expected to recognize a quadratic equation and its shape, and know how to find its vertex and intercepts. Students may be asked to graph other polynomial functions by substitution. The hardest application problems for students are typified by the following: *Graph the quadratic equation* $y = 8 - x - x^2$ *; find its vertex and x-intercepts.* *The height H, in feet, of a projectile with an initial velocity of 96 ft./sec. is given by the equation* $H = -16t^2 + 96t$ *, where t = time in seconds. Use the graph of this equation, which is given above (not*	Graphing of quadratic equations is extended to include recognizing when a quadratic equation has complex solutions. Students should be asked to recognize the shape of other polynomial function, in particular cubic and quartic equations, and identify its maximum number of roots. In addition, they may be asked to find the x-intercepts for some cubic and quartic equations by factoring. Graphing calculators may be used to estimate intercepts and max/min points. Graphing of rational functions shall be introduced along with the concept of asymptotes. Use of a calculator to fit a quadratic, cubic or quartic equation to a data set may be required. The hardest application problems for students are typified by the following: *Graph the following functions, finding approximate and exact values (if possible) for the x- and y-intercepts. Determine the multiplicity of the roots, the power function the graph resembles for large values of x, the number of turning points and any asymptotes. Estimate all local maxima and minima.* $f(x) = (x-1)^2(x+3)(x+1)$, $R(x) = \dfrac{x^2+x-6}{x^2-x-6}$ *, and* $f(x) = 12x^3 + 39.8x^2 - 4.4x - 3.4$ *Explain how you tell from its equation that a polynomial is a parabola.*

	shown here), to find the following. *a. How many seconds after the launch is the projectile 128 ft. above the ground?* *b. What is the projectile's maximum height and when does it reach that height?* *c. How many seconds after the launch does the projectile return to the ground?*	*The height H, in feet, of a projectile with an initial velocity of 96 ft./sec launched from 120 ft. above ground level is given by the equation* $H = -16t^2 + 96t + 120$, *where t = time in seconds. Sketch the graph of this function and find the following.* *a. How many seconds after the launch is the projectile 128 ft. above the ground?* *b. What is the projectile's maximum height and when does it reach that height?* *c. How many seconds after the launch does the projectile return to the ground?*
Circles & other Conic Sections	Circles and conic sections are not usually covered in Introductory Algebra.	Graphing of circles and finding their center and radius are introduced. In addition, parabolas, ellipses, and hyperbolas may be introduced. The hardest application problems for students concerning circles are typified by the following: *Find the center and radius of the following circle and sketch its graph:* $2x^2 + 2y^2 - 12x + 8y - 24 = 0$.
Equations Containing Radicals	Equations containing radicals are not usually covered in Introductory Algebra.	Equations containing radicals are often introduced in Intermediate Algebra. The hardest problems for students concerning equations containing radicals are typified by the following: *Use a graphing calculator to estimate the real solutions to the following equations and, if possible, find the exact solutions algebraically.* $\sqrt[3]{2x-4} = 2$, and $\sqrt{2x+3} - \sqrt{x+2} = 2$
Function Notation	In Introductory Algebra, if function notation is introduced, it is usually limited to substituting values of x into equations such as $f(x) = x^2 + 2x - 5$.	Functions are defined, along with function notation. The concept of domain and range are introduced. Composition of functions is also defined. The hardest problems for students concerning function notation are typified by the following: *Find the domain and range of the function* $f(x) = \sqrt{x^2 + x - 6}$. *Find* $f \circ g(x)$ *if,* $f(x) = \frac{x-1}{x+3}$ *and* $g(x) = \frac{x}{2-x}$.
Families of Functions (Symmetry, Shifts, & Stretching)	Families of functions are not usually covered in Introductory Algebra.	Families of functions such as $y = f(x) + c$, $y = f(x+c)$, and $y = f(kx)$ are introduced. Symmetry about the *x-axis, y- axis* and origin is also usually covered.
Inequalities	In Introductory Algebra, if inequalities are covered, they are usually limited to graphing the solution of one or more linear inequalities of the type: $y \geq 2x + 5$.	In intermediate algebra students are often asked to solve quadratic and rational inequalities of the type $f(x) > 0$, $f(x) \geq 0$, $f(x) < 0$ and $f(x) \leq 0$, and to graph inequalities of the type $y > f(x), y \geq f(x), y < f(x)$ and $y \leq f(x)$ for functions such as: $f(x) = x^2 + 2x - 6$, *and* $f(x) = \frac{1}{x-1} + \frac{2}{7x-4}$. Note: sometimes these skills may be asked for indirectly in problems such as ones asking student to find the domain and range of the functions, expressing their answers in interval notation.
One-to- One Functions & Inverse Functions	One-to-one functions & inverse functions are not usually covered in Introductory Algebra.	Students should be able to recognize one-to-one functions, find the graph of their inverse as the reflection across the line $y = x$, and find their inverse algebraically. Students should also understand the relationship between the domain and range of a function, and that of its inverse function. The hardest problems for students concerning inverse functions are typified by the following:

		Find the inverse, $f^{-1}(x)$, *of the function* $f(x)=\dfrac{2x-5}{x+3}$. *Use interval notation to state the domain and range of both* $f(x)$ *and* $f^{-1}(x)$. *In a short paragraph, explain the procedure you should use to find the inverse of a function.*
Absolute Value & Greatest Integer Functions	The concept of absolute value, along with graphing functions such as, $y=\lvert x-3\rvert$, is sometimes introduced in Introductory Algebra, but not the greatest integer function.	In Intermediate Algebra, students may be asked to solve both graphically and algebraically absolute value equations or inequalities, such as, $\lvert 3x-2\rvert \le 0.02$. The greatest integer function is sometimes introduced in Intermediate Algebra.
Exponential & Log Functions	Exponential and log functions are not usually covered in Introductory Algebra.	Exponential functions are usually introduced in Intermediate Algebra. Log functions are defined as the inverse of the exponential function; however, the laws of logarithms are not usually stressed. Students are expected to recognize both types of functions and to graph them. The hardest problems for students concerning exponential and log functions are typified by the following: *A model for the number of people N in a community college who have heard a certain rumor is* $N=P\left[1-e^{-0.15d}\right]$, *where P is the total population of the community college and d is the number of days that have elapsed since the rumor began. In a community of 1000 students, find the following:* *a. How many students will have heard the rumor after 3 days?* *b. How many days will have elapsed before 450 students have heard the rumor?* *The model year and prices of used Honda Accords are given in the table below (not shown). Draw a scatter plot of this data and use your calculator to find the equation of the curve that best fits this data. (Note to the reader: the data fits an exponential function.)*

SOLVING EQUATIONS AND INEQUALITIES

	Introductory Algebra	**Intermediate Algebra**
Linear Equations	The notion of an algebraic equation is introduced (2x – 3 = 6) along with a solution set. Properties of equality are discussed along with the addition property (a = b then a + c = b + c) and the multiplication property (a = b then ac = bc). Method of solving is the focus. Variations include equations with parenthesis and fractional coefficients. Application problems are discussed primarily consecutive Integer, categories and geometry problems involving rectangles. The hardest type problems might include: **1.** *3x – 8 + 2x = x – 6 + 10* **2.** *3(x + 7) + 2 = 4(x – 3) + 5x* **3.** *(1/2)x – ¾ = (1/3)x + 1/6* **4.** *Two consecutive integers sum up to 85. Find the integers.* **5.** *An audience of 300 students has*	The topics from Introductory Algebra are reviewed and extended in a condensed format. The types of problems given are of a more complex nature. Work problems with more in depth reasoning are focused on. Typical examples: *In a class of 62 students, the number of females is one less than twice the number of males. How many females and how many males are there in the class?* *A sum of $10,000 is split between 2 investments, one pays 9%, another pays 11%. If the return of the 11% investment is 60 dollars more per year than the 9%, how much is in each fund?*

	two times as many females as males. Find how many males and how many females were in the audience.	
Literal Equations	The notion of formula or literal equations is introduced – generally in the context of geometric figure. Plugging in or evaluating formulas is examined along with solving for a given letter in simple situations, such as, solving 2l + 2w = P for l. Typical problems: *Find the area of a circular region if the circumference is12p units.*	Equations that contain more than one variable are referred to as literal equations. Sometimes referred to as formulas. Practice is given in solving for a given variable, i.e., solving $E = mc^2$ for m. Sample application problems would be: *How many gallons of a 15% salt solution need to be mixed with a 35% salt solution to obtain 8 gallons of 30% salt solution?*
Variation (Direct & Inverse)	Very few introductory algebra texts have much on direct or inverse variation as a distinct topic.	Most texts have a section or unit or direct and inverse variation or proportionality between two variables, and they discuss the constant of proportionality application problems. They include problems from all areas of science. Typical examples of problems students find the hardest are: *If A varies jointly as b & h. If A = 120, when b = 6 and h = 5, find A when b = 12 and h = 10.* *The volume of gas (V) varies directly as temperature (T) and inversely as pressure (p). Set up equation and evaluate the constant of proportionality of V = 48 when T = 320 and P= 20.*
Quadratic Equations	Quadratics are introduced as part of introductory algebra usually toward the end of the course. Solutions initially come from the use of ab = 0 and factoring. Some introductory algebra courses cover completing the square and quadratic formula. The most difficult problems encountered are typified by: *A piece of wire 56 inches long is cut into 2 pieces and each piece is bent into the shape of a square. The sum of the areas of the 2 squares is 100 square inches, find the length of each piece of wire.*	The focus of each method is reviewed. A discussion of the discriminate to predict the number of solutions is generally gone over and the applications go into more depth. Quadratic equations with complex number solutions are usually introduced. The most difficult application problem encountered are typified by: *A 62' wire that makes an angle of 60 degrees with the ground is attached to a telephone pole. Find the distance from the base of the pole to the point on the pole where the wire is attached. Express your answer to the nearest 10^{th} of a foot.*
Inequalities	Methods of solving very basic inequalities are covered (2x – 8 < 3x + 10) and tied in with solving basic equations. Occasionally compound inequalities are covered. Typical examples: *2(x – 8) + 9(x + 2) < 25* *Betty has earned a score of 96, 90 and 94 on the first 2 exams in a class. There are 2 more exams and she wants an overall average of 92 or better. What must the average on the last two exams be to achieve this?*	A review of the basic techniques is covered. All topics are extended and they also involve absolute value, quadratic an rational inequalities. Occasionally linear inequalities in two variables are covered. Interval notation is usually introduced. The graphing calculator is often used to aid in finding the solutions. These problems may be the ones that students find the most difficult, because they combine many separate skills into one multi-step problem. They require perhaps the greatest amount of synthesis of any problem requiring only manipulative skills. The most difficult problems encountered are typified by: *Solve the following inequalities, expressing your answer in interval notation.* $(x + 2)(x - 7) < 0$ $x^2 + 2x - 7 > 0$ $\|7x - 6\| < 22$ $\frac{1}{x-2} < \frac{2}{3x-9}$

Equations with Radicals	Very simple equations with the variable under the square root may be presented, along with the technique to solve the equation, for example, $\sqrt{x-2}-5$.	What has been covered in introductory algebra is reviewed, and the techniques for solving equations containing radicals are furthered by investigating some more complicated situations. Typical problems: $\sqrt{2x+3}+7=x+2$ $\sqrt{x+1}+2=\sqrt{x+2}$ *Solve for w:* $T=2p\sqrt{\frac{w}{40}}$.
Rational Equations	Rational equations are usually introduced. Typical problems include: $\frac{x}{6}-\frac{x}{10}=\frac{1}{6}$ $\frac{5}{x-1}=\frac{3}{x+2}$ $\frac{x}{x+4}-\frac{4}{x-4}=\frac{x^2+16}{x^2-16}$	This topic is usually reviewed in Intermediate Algebra as part of a general review to solving equations that usually includes linear, quadratic and rational equations together. Further synthesis may be stressed by investigating what happens as $n\to 0$ in the equation $\frac{5}{n}+\frac{1}{2}=\frac{9}{n}$. Typical examples: $\frac{x}{x-2}+\frac{2}{3}=\frac{2}{x-4}$ *Dan agreed to mow a vacant lot for $12. It took him an hour longer than what he had anticipated, so he earned $1.00 per hour less than he had originally calculated. How long had he anticipated that it would take him to mow the lot?*
Absolute Value Equations	This topic is not always covered at the introductory level – usually relegated to the additional topics chapter – and when covered, only the simplest of type of linear absolute value equations, like $\|x\| = 3$ or $\|2x - 1\| =7$ are covered. The number line is generally used to solve inequalities, such as $\|2x + 1\|> 1$ $\|x\| < 3$ or $\|2x - 1\|<5$, and applications are rarely given. The hardest type of problems encountered are: $\|3x + 1\| =13$ $\|5x + 3\|<14$	A much fuller discussion of absolute value equations and inequalities are dealt with here on this level. This Distance from 0 idea is usually the major application. Typical examples: $\|2x + 1\| > 1$ $\|x + 4\|<0$
Systems of Equations	After introducing the Cartesian coordinate system linear equations in two variables are solved by three methods – graphing, elimination and substitution. No linear algebra techniques or system beyond a 2x2 system of equations is usually covered. Example of a harder applications: *The sum of two numbers is 52. The larger number is two more than four times the smaller number. Find the numbers.* *Find the solution to* $\begin{matrix}2x+3y=31\\3x+5y=-20\end{matrix}$	Usually the basic techniques are reviews and then expanded to 3 x 3 systems. Some intermediate algebra courses use linear algebra (matrices and determinants) approaches to solutions. Typical example: *Solve by any method:* $2x-y+3x=-10$ $x+2y-3z=2$ $3x+2y+5z=-16$

Exponential & Log Equations	This topic is not usually covered in Introductory Algebra.	Exponential equations are often introduced in Intermediate Algebra. Logs are used to find solutions for variables that are in the exponent of an exponential equation. In most application problems the exponential equation is explicitly given. The hardest application problems are typified by: *The annual profit P of a company due to the sales of a particular item after it has been on the market for x years is determined to be* $P = \$100{,}000 - \$60{,}000\left(\frac{1}{2}\right)^x$ *a. What is the profit after 5 years? (10 years?)* *b. When will the profit be \$80,000?* *c. What is the most profit that the company can expect from this product?*
Manipulative Skills not Implied by the Previous Topics		
	Introductory Algebra	**Intermediate Algebra**
Exponents	Positive and negative exponents are usually covered in the first half of Introductory Algebra. Typical difficulty of problems. $a^{-11}a^{-3}a^{-7}$ $\frac{x^{-2}}{x^{-7}}$	Facility with exponents is assumed of students entering Intermediate Algebra, so this topic is usually reviewed as it is needed in other problems. If negative exponents are not covered in Introductory Algebra, they are covered here. Problems tend to be more complex and rational exponents are usually included. The complexity of the problems is typified by: $\left[\frac{9x^2y^{1/3}}{x^{1/3}y}\right]^{1/2}$ $\frac{(2x+5)^{1/3}(2x+5)^{-1/2}}{(2x+5)^{-3/4}}\ 0$
Polynomial Long Division	The long division axiom for polynomials is usually introduced in Introductory Algebra. Typical example: $(x^2+4x-14)\div(x+6)$	In Intermediate Algebra polynomial long division is often reviewed as part of a more complex task, such as finding oblique asymptotes for rational functions. Typical example: *Find the horizontal and oblique asymptotes to* $H(x) = \frac{3x^4 - x^2}{x^3 - x^2 + 1}$.
Factoring	Traditionally a lot of time is spent on trinomial factoring in Introductory Algebra. With the advent of graphing calculators, the time spent on this topic may be reduced or incorporated into finding roots of equations.	Factoring of trinomials is usually assumed of students entering Intermediate Algebra. In is usually reviewed in the context of other problems, such as, solving quadratic equations.