

**Sample Course: Finite Mathematics** (Prepared by Daniel Symancyk 10/16/02)

**Sample Catalog Description:**

An introduction to finite mathematics. Topics include systems of linear equations, matrices, the Gauss-Jordan method, inequalities and linear programming, sets and counting techniques, probability, difference equations, Markov processes and game theory. Applications to economics, business and social science are discussed.

**Learning Objectives:**

At the end of this course, student should be able to:

- A. Solve applications involving matrices.
- B. Solve linear programming problems using the graphical approach.
- C. Use set theory, the principles of counting and probability concepts to calculate probabilities.
- D. Apply Markov processes to the solution of probability problems.
- E.. Model interest and finance applications using difference equations.
- F. Apply elementary game theory to produce optimal strategies in games.
- G. Apply the simplex method. (optional)

**Brief Description of Applications Involved by Objective**

(More detailed sample problems are given later in this document.)

Objective A includes working with matrix operations , procedures for finding the inverse of a matrix when it exists and the solution of matrix equations. Applications would include the Leontief Input-Output economic model and simple coding theory problems.

Objective B includes the optimization of linear functions in two variables subject to linear constraints in two variables using graphical methods. Typical applications in business allow one to determine how to maximize revenue or minimize cost taking into account scarce resources.

Objective C includes counting techniques and the calculation of basic and conditional probabilities used in decision making processes.

Objective D uses probability and matrices to model Markov processes. For example, suppose that 70% of the daughters of women who work outside the home work outside the home and that 20% of daughters of women who do not work outside the home work outside the home. In a county in which 55% of women work outside the home, find the percentage of women who work outside the home in three generations.

Objective E includes compound interest, amortization, and annuities using difference equations.

Objective F introduces using matrices and probability to find effective strategies that can be used in competitive situations.

Objective G includes the optimization of linear functions in two or more variables subject to linear constraints in two or more variables using matrices and the simplex method.

## Sample of College-Level Math Attributes in Finite Mathematics/Math Modeling

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*Note: Examples reference Maryland General Education Attributes, objectives in sample Finite Math course, and pre-requisite skills from Intermediate Algebra.*

<b>Maryland Attribute(s)</b>	<b>Examples Found in Finite Math/Math Modeling Courses</b>	<b>Intermediate Algebra Background</b>
1, 3, 6	<p>A utility company produces electricity and water. To produce each dollar worth of electricity the company must spend \$0.15 on electricity and \$0.40 on water. In addition, to produce each dollar worth of water the utility must spend \$0.25 on electricity and \$0.10 on water.</p> <p>A.) Write out the input-output table. Label the rows and columns appropriately.</p> <p>B.) If the company must meet external demands of \$150 of electricity and \$40 of water each month per household, find the gross production that meets this demand.</p> <p>Requires objective A.</p>	<p>Solve <math>a - ax = d</math> for <math>x</math>.</p> <p>Solving literal equations using additive and multiplicative inverses. Since the input-output problem involves a matrix equation, there is an added level of complexity since matrix multiplication is not commutative and it is not a trivial matter to find a multiplicative inverse when one exists.</p>

<p>1, 3, 5, 6</p>	<p>A cabinet company produces two styles of computer tables. The company has enough raw material to produce up to 100 tables. Each larger table requires 4 hours for construction while it takes only 3 hours for each smaller table. The company's profit is \$45 per large table and \$36 for each small table. Due to existing orders, the company must produce at least 2 large tables and 3 small tables. If the company has 40 hours available, how many of each type table should be made to maximize the company's profit?</p> <p>Requires objective B.</p>	<p>A cabinet company produces two types of tables, large and small. Each large table requires 4 hours for construction while each small table takes only three hours to make. If there are up to 60 hours available to work on tables, write an inequality that describes the situation and graph the inequality. Graph the solution to the inequality under the assumption that the number of each type of table cannot be negative.</p> <p>Translate into linear expressions and linear inequalities. Uses the graphical solution to systems of linear inequalities. Recognition of the corner points of the feasible region as solutions to linear systems and then solving the linear systems is critical to finding the vertices.</p>
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<p>1, 3, 5, 6</p>	<p>A company has two warehouses and two stores. The first warehouse has 40 DVD players while the second one has 100 DVD players. Store #1 needs 30 DVD players and store #2 needs 10 DVD players. The transportation costs per DVD player are given below:</p> <p style="text-align: center;"> \$8 from warehouse #1 to store #1  \$4 from warehouse #2 to store #1  \$6 from warehouse #1 to store #2  \$2 from warehouse #2 to store #2 </p> <p>What is the most economical way to supply the DVD players?</p> <p>Requires objective G.</p>	<p>A cabinet company produces two types of tables, large and small. Each large table requires 4 hours for construction while each small table takes only three hours to make. If there are up to 60 hours available to work on tables, write an inequality that describes the situation and graph the inequality. Graph the solution to the inequality under the assumption that the number of each type of table cannot be negative.</p> <p>Translate into linear expressions and linear inequalities. Recognize that because of the number of variables, the graphical method can't be used in the DVD problem and a matrix method must then be used.</p>
<p>1, 3, 5, 6</p>	<p>Minimize  <math>f = x + y + 3z</math>  subject to constraints of  <math>x + 6y \leq 10</math>  <math>x - y \leq 6</math>  <math>y + 2z \geq 8</math>  <math>x \geq 0, y \geq 0, z \geq 0</math></p> <p>Requires objective G.</p>	<p>Find the minimum value of <math>f</math> when <math>f = -0.4x^2 + 3x + 7</math></p> <p>(Note: while the finite math problem given involves a linear function, this type of quadratic is typical of the Intermediate Algebra problem where minimize and maximize are first introduced.)</p> <p>Recognize that because of the number of variables a matrix method rather than a graphical method is needed.</p>

1, 2, 3, 4	<p>Thirty-five community college students were surveyed about what classes they were taking.</p> <p>Four students said they were taking art, biology, and chemistry.</p> <p>Six were taking art and biology.</p> <p>Ten were taking art and chemistry.</p> <p>Five said they were in biology and chemistry.</p> <p>17 were in art.</p> <p>10 were in biology.</p> <p>20 were in chemistry.</p> <p>A.) How many were taking biology only?</p> <p>B.) How many were taking none of the three subjects?</p> <p>Requires objective C.</p>	<p>Solve <math>2x + 1 \leq 5</math> or <math>-2x &gt; 3</math></p> <p>Solve <math>2x + 1 \leq 5</math> and <math>-2x &gt; 3</math></p> <p>Solve <math> -x+1  \leq 7</math></p> <p>Solve <math> -x+1  &gt; 7</math></p> <p>Recognize the connection between “and”, “or”, and “not” with set operations. This typically surfaces in Intermediate Algebra when solving compound inequalities and absolute value inequalities.</p>
3, 4, 5	<p>How many arrangements of six letters begin with a vowel and end with a different vowel if</p> <p>A.) There are four distinct consonants in between the distinct vowels?</p> <p>B.) There are four consonants in between the distinct vowels?</p> <p>Requires objective C.</p>	<p>No real direct connection with Intermediate Algebra.</p> <p>Evaluate <math>(5)(21)(20)(19)(18)(4)</math></p> <p>Evaluate <math>(5)(21)^4(4)</math></p> <p>Organize information using a picture is helpful when first introducing problems using the fundamental counting principle.</p>
3, 4, 5	<p>A company interested in doing business in China wants to send a delegation of five company officials on a visit to China. Of the twelve officials with the proper visas, seven are male and five are female. How many delegations with three men and two women are possible?</p> <p>Requires objective C.</p>	<p>Find the coefficient of <math>x^4 y^3</math> in <math>(x + y)^7</math></p> <p>Find the coefficient of <math>x^3 y^2</math> in <math>(x + y)^5</math></p> <p>Pascal’s triangle and the binomial coefficients</p>

1, 3, 4, 5	<p>An independent laboratory tested a total of 200 tomato seeds from two companies, ActiveGrow and BioBlume. 150 of these seeds germinated. Of those that germinated, 70 came from BioBlume. If 20 of the non-germinating seeds came from ActiveGrow, find the probability that the seed came from ActiveGrow given that germination occurred.</p> <p>Requires objective C.</p>	<p>“Orange-Thirst” is 20% orange juice and “Quencho” is 10% orange juice. How many liters of each should be mixed together to get 10 liters of a mixture that is 17% orange juice?</p> <p>While the finite math question given here does not use a system of equations as you might in solving the juice problem, the skill learned in Intermediate Algebra of organizing information into a table is helpful in this probability question.</p>
1, 2, 3, 4, 5	<p>Assume that E and F are events with <math>P(E) = .3, P(F) = .7, P(E \cup F) = .8</math></p> <p>A.) Translate <math>P(E \cap F)</math> into words and evaluate it.</p> <p>B.) Translate <math>P(E F)</math> into words and evaluate it.</p> <p>C.) Translate <math>P(E')</math> into words and evaluate it.</p> <p>D.) Find <math>P(E \cap F')</math></p> <p>Requires objective C.</p>	<p>Solve <math>D = A + B - x</math> for x.</p> <p>Solve <math>P = AB</math> for B.</p> <p>Solve <math>A + B = 1</math> for A.</p> <p>In addition to Intermediate Algebra problems dealing with compound inequalities which reinforce the connections between “and”, “or”, and “not” with set operations, this type of problem requires solving literal equations.</p>
1, 3, 4, 5	<p>Four of 20 bulbs in a box are defective. A sample of three is randomly selected. Find the probability that</p> <p>A.) the sample contains no defective bulbs.</p> <p>B.) the sample contains at least one defective fuse.</p> <p>Requires objective C.</p>	<p>Let <math>S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}</math>. List the elements in set S that are at least 2.</p> <p>Connection between logic and set operations.</p>

<p>1, 2, 3, 4</p>	<p>Let <math>A = \begin{bmatrix} .3 &amp; .5 \\ .7 &amp; .5 \end{bmatrix}</math>, <math>V = \begin{bmatrix} .45 \\ .55 \end{bmatrix}</math>, and <math>W = \begin{bmatrix} .9 \\ .1 \end{bmatrix}</math></p> <p>For the regular stochastic matrix <math>A</math> and initial distributions <math>V</math> and <math>W</math>, find</p> <p><math>AV, A^2V, A^3V</math>, and <math>A^{10}V</math></p> <p><math>AW, A^2W, A^3W</math>, and <math>A^{10}W</math></p> <p>What appears to be happening?</p> <p>Requires objectives D and A.</p>	<p>Let <math>f(x) = 4\left(\frac{2}{7}\right)^x</math> Find</p> <p><math>f(0), f(1), f(2), f(3), f(10)</math>, and <math>f(100)</math></p> <p>What happens to <math>f(x)</math> when <math>x</math> is extremely large?</p> <p>Creating a table of values for a function to introduce important characteristics of the function.</p>
<p>1, 3, 5, 6</p>	<p>Find the long term trend in the situation described below:</p> <p>In a manufacturing process, the state of the assembly line is monitored hourly as either being ahead of schedule, on schedule, or behind schedule. If the line is on schedule the probability that it will be ahead of schedule the next hour is 0.25, on schedule is 0.40, and behind schedule is 0.35. If the line is ahead of schedule the probabilities are 0.15, 0.15, and 0.70 for ahead of schedule, on schedule, and behind schedule, respectively. If the line is behind schedule, the probabilities are 0.30, 0.60, and 0.10 for ahead of schedule, on schedule, and behind schedule, respectively.</p> <p>Requires objectives D and A.</p>	<p>The WKS corporation produces three styles of hats, model #9, #11, and #13.</p> <p>There is enough material to make 90 hats. Each #9 requires 2 minutes in department A and 2 minutes in department B. Each #11 requires 3 minutes in A and 3 minutes in B. Each #13 takes 3 minutes and 4 minutes in the respective departments. If A is available for 220 minutes and B is available for 240 minutes, how many of each type of hat can be made?</p> <p>Organize information into a table. Solve systems of linear equations. The finite math problem given involves working with 4 linear equations in 3 variables.</p>

<p>1, 3, 5, 6</p>	<p>Two competing cable TV companies (one called R and one called C) would like to increase their market shares. A study has shown that</p> <p>If R lowers its rates and so does C, then R will get 60% of the market.</p> <p>If R lowers its rates and C does not, then R will get 35% of the market.</p> <p>If R does not lower its rates and C does, then R will get 80% of the market.</p> <p>If R does not lower its rates and C does not lower its rates, then R will get 50% of the market.</p> <p>What should each company do in order to maximize its market share?</p> <p>Requires objective F.</p>	<p>“Orange-Thirst” is 20% orange juice and “Quencho” is 10% orange juice. How many liters of each should be mixed together to get 10 liters of a mixture that is 17% orange juice?</p> <p>While the finite math question given here does not use a system of equations as you might in solving the juice problem, the skill learned in Intermediate Algebra of organizing information into a table is helpful in setting up this game theory question.</p>
<p>1, 2, 3, 5, 6</p>	<p>The population <math>y</math> of a country is currently 100 million but is declining at the rate of 1% per year due to an excess of deaths over births. In addition, the country is gaining 2 million people per year due to immigration.</p> <p>A.) Find <math>y_0</math>, <math>y_1</math>, and <math>y_2</math>.</p> <p>B.) Find the difference equation.</p> <p>Requires objective E.</p>	<p>Assume that the number of record players sold by a company has been declining at a rate of 750 per year since 1977 when 24000 were sold.</p> <p>A.) Find the number sold in 1978, 1979, and 1980.</p> <p>B.) Find an equation for <math>y</math>, the number sold, <math>x</math> years after 1977</p> <p>Translating expressions into equations. Evaluating equations. Understanding the linear model.</p>

<p>1, 2, 3, 5, 6</p>	<p>Assume that the daily increase in the number of people who have heard a rumor (perhaps the one that your local community college is going to become a four year school) is proportional to the number of people in the region who have not heard the rumor. If there are 5 million people in the region and the proportionality constant is .02, find the difference equation.</p> <p>Requires objective E.</p>	<p>If <math>y</math> varies directly with the square of <math>x</math> and <math>y</math> equals 8 when <math>x</math> equals 6, find the equation for <math>y</math> in terms of <math>x</math>.</p> <p>Translating expressions into equations. Working with variation.</p>
<p>1, 2, 3, 5, 6</p>	<p>The Jones have \$6000 in an account paying 4% compounded monthly. They plan on depositing \$250 into the account at the end of the month for 5 years.</p> <p>A.) Find a difference equation that describes this situation.</p> <p>B.) Solve the difference equation and find the amount in the account in 5 years.</p> <p>C.) Make a graph showing the amount in the account at the end of each month for 60 months.</p> <p>Requires objective E.</p>	<p>Vera invested a total of \$10,000. Part of this was invested at 5% and the rest at 6%. If the total simple interest earned over one year was \$540, find the amount invested at each rate.</p> <p>Understand simple interest. Translating expressions into equations. Simplifying equations involving rational expressions that show up when solving difference equations. Evaluating equations. Graphing equations.</p>

<p>1, 3, 5</p>	<p>For each difference equation</p> <p>a.) Determine if the solution is monotonic or oscillating</p> <p>b.) If the solution is monotonic, determine whether it is increasing or decreasing.</p> <p>c.) Determine if the solution is asymptotic or unbounded.</p> <p>d.) Make a graph of the solution.</p> $y_{n+1} = 1.5y_n + 2, y_0 = 100.$ $y_{n+1} = -1.5y_n + 2, y_0 = 100.$ $y_{n+1} = 0.5y_n + 1, y_0 = 6.$ $y_{n+1} = -0.5y_n + 1, y_0 = 6.$ <p>Requires objective E.</p>	<p>Determine whether the following are increasing or decreasing:</p> $y = -3x + 2$ $y = 4.3x - 17$ $y = \left(\frac{2}{3}\right)^x$ $y = \left(\frac{3}{2}\right)^x$ <p>Understand when linear functions and basic exponential functions are increasing or decreasing.</p>
<p>1, 3, 4, 5, 6</p>	<p>Suppose that Mary Brown can afford to pay \$800 per month and the interest rate is 6% compounded monthly. How much money can she borrow for 15 years? Justify your answer.</p> <p>Requires objective E.</p>	<p>Solve <math>0 = \frac{b}{1-a} + \left(y - \frac{b}{1-a}\right)k</math> for y.</p> <p>Understand simple interest. Translating expressions into equations. Simplifying equations involving rational expressions that show up when solving difference equations. Solving rational equations.</p>