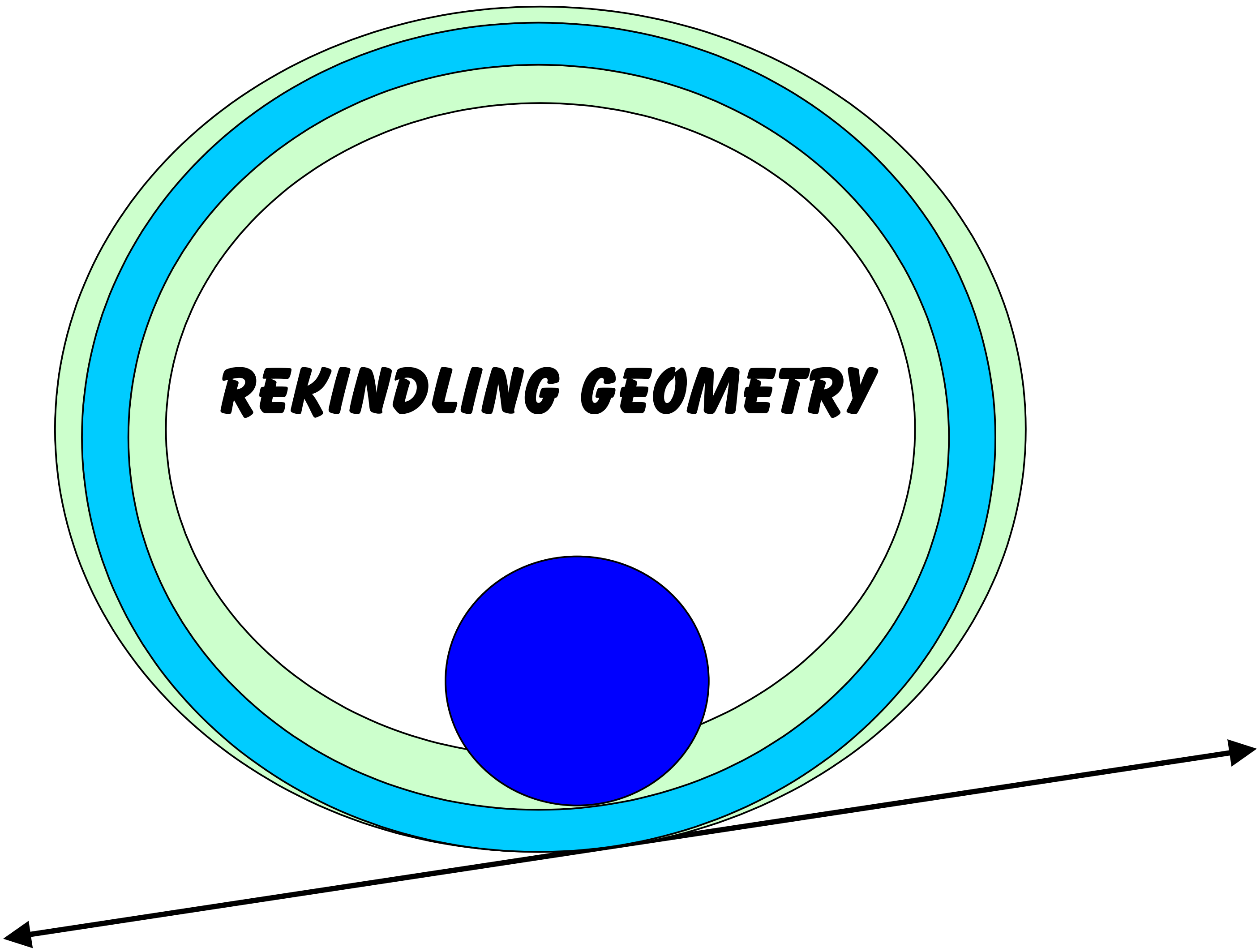

REKINDLING GEOMETRY

29TH AMATYC ANNUAL CONFERENCE

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“Studying geometry benefits students in a number of ways. Geometry enables students to represent and make sense of the world, analyze and solve problems and represent abstract symbols pictorially to facilitate understanding”

Source: NCTM, 2000

“To help students learn something new (construct new neural networks), we must always start a new unit by stimulating each student’s knowledge/neural networks that are related in some way to the new skill or knowledge. This is essential because brain structures (dendrites, spines, networks) cannot grow in a void.”

Increasing these brain structures (dendrites, spines, networks) “occurs only when the learner is an active learner.”

Source: Natural Teaching Method Based on Learning Theory, by Rita Smilkstein

The AMATYC Standards reinforce the need for active rather than passive learning. They encourage us to "engage in exploratory activities that will lead students to form conjectures."

CROSSROADS IN MATHEMATICS
Internet Link: www.imacc.org/standards/

The Standards for Intellectual Development address desired modes of student thinking and represent goals for student outcomes.

Standard 1-1: Problem Solving
: **Students will engage in substantial mathematical problem solving.**

Standard 1-2: Modeling
: **Students will learn mathematics through modeling real-world situations.**

Standard 1-3: Reasoning
: **Students will expand their mathematical reasoning skills as they develop convincing mathematical arguments.**

Standard 1-4: Connecting with Other Disciplines
: **Students will develop the view that mathematics is a growing discipline, interrelated with human culture, and understand its connections to other disciplines.**

Standard 1-5: Communicating
: **Students will acquire the ability to read, write, listen to, and speak mathematics.**

Standard 1-6: Using Technology
: **Students will use appropriate technology to enhance their mathematical thinking and understanding and to solve mathematical problems and judge the reasonableness of their results.**

Standard 1-7: Developing Mathematical Power
: **Students will engage in rich experiences that encourage independent, nontrivial exploration in mathematics, develop and reinforce tenacity and confidence in their abilities to use mathematics, and inspire them to pursue the study of mathematics and related disciplines.**

The *Standards for Content* provide guidelines for the selection of content that will be taught at the introductory level.

Standard C-1: Number Sense
: **Students will perform arithmetic operations, as well as reason and draw conclusions from numerical information.**

Standard C-2: Symbolism and Algebra
: **Students will translate problem situations into their symbolic representations and use those representations to solve problems.**

Standard C-3: Geometry
: **Students will develop a spatial and measurement sense.**

Standard C-4: Function
: Students will demonstrate understanding of the concept of function by several

means (verbally, numerically, graphically, and symbolically) and incorporate it as a central theme into their use of mathematics.

Standard C-5: Discrete Mathematics

Students will use discrete mathematical algorithms and develop combinatorial abilities in order to solve problems of finite character and enumerate sets without direct counting.

Standard C-6: Probability and Statistics

Students will analyze data and use probability and statistical models to make inferences about real-world situations.

Standard C-7: Deductive Proof

Students will appreciate the deductive nature of mathematics as an identifying characteristic of the discipline, recognize the roles of definitions, axioms, and theorems, and identify and construct valid deductive arguments.

The *Standards for Pedagogy* recommend the use of instructional strategies that provide for student activity and interaction and for student-constructed knowledge.

Standard P-1: Teaching with Technology

Mathematics faculty will model the use of appropriate technology in the teaching of mathematics so that students can benefit from the opportunities it presents as a medium of instruction.

Standard P-2: Interactive and Collaborative Learning

Mathematics faculty will foster interactive learning through student writing, reading, speaking, and collaborative activities so that students can learn to work effectively in groups and communicate about mathematics both orally and in writing.

Standard P-3: Connecting with Other Experiences

Mathematics faculty will actively involve students in meaningful mathematics problems that build upon their experiences, focus on broad mathematical themes, and build connections within branches of mathematics and between mathematics and other disciplines so that students will view mathematics as a connected whole relevant to their lives.

Standard P-4: Multiple Approaches

Mathematics faculty will model the use of multiple approaches-numerical, graphical, symbolic, and verbal-to help students learn a variety of techniques for solving problems.

Standard P-5: Experiencing Mathematics

Mathematics faculty will provide learning activities, including projects and apprenticeships, that promote independent thinking and require sustained effort and time so that students will have the confidence to access and use needed mathematics and other technical information independently, to form conjectures from an array of specific examples, and to draw conclusions from general principles.

Standards for Intellectual Development

Standard I-2: Modeling

Students will learn mathematics through modeling real-world situations.

Standard I-3: Reasoning

Students will expand their mathematical reasoning skills as they develop convincing mathematical arguments.

Standards for Content

Standard C-3: Geometry

Students will develop a spatial and measurement sense.

Standards for Pedagogy

Standard P-1: Teaching with Technology

Mathematics faculty will model the use of appropriate technology in the teaching of mathematics so that students can benefit from the opportunities it presents as a medium of instruction.

AMATYC Intellectual Development Standard
I-2: Modeling

Investigation of the Lateral Area of a Cylinder

Materials needed: empty cardboard tube from a roll of gift wrap, paper towels or toilet paper, scissors, ruler. Do this activity using teams of 4 people. Assign each team member one of the following tasks.

- Measure the diameter and length of the tube to the nearest eighth of an inch.
- Calculate the lateral area of the cylinder. Round the answer to the nearest whole number.
  Remember: LA of cylinder $= 2\pi rh$
- With a pair of scissors, cut along the "glue line" of the tube and open it. Name the shape of the figure.
- Find the area of this figure. Round the area to the nearest whole number.

How does the area of the two dimensional figure compare to the lateral area of the cylinder? Is this coincidence? Explain.

AMAYTC Intellectual Development Standard
I-3: Reasoning

AAA Congruency Theorem

For this activity a ruler and protractor will be needed. Do the activity in groups of 3 or 4.

- Using a protractor, draw a triangle with three angles that measure $30°, 60°$ and $90°$.
- Compare your triangle with at least two other people. Are the triangles congruent?

Make a conjecture about an AAA theorem of congruence? Explain your reasoning.

<u>AMAYTC Intellectual Development Standard</u>
<u>I-3: Reasoning</u>

Circle Puzzle

Given: $m\angle AOB = 55°$; ΔOBC is equilateral;
$\overline{OA}, \overline{OB}, \overline{OC}$ and $\overline{OD}$ are radii of circle *O*.

Find:

a) $m\angle BCD$

b) $m\overset{\frown}{AB}$

c) $m\angle ADB$

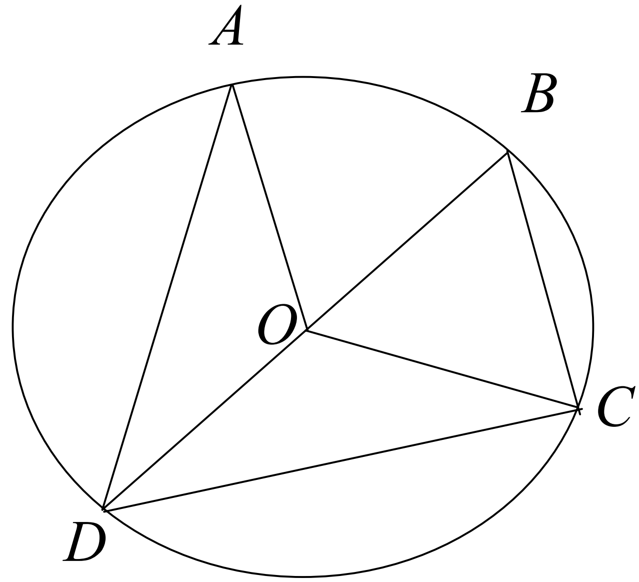
d) $m\overset{\frown}{BC}$

e) $m\overset{\frown}{DC}$

f) $m\angle DOC$

g) $m\angle ODC$

h) $m\overset{\frown}{AD}$



Some of the ideas needed for circle puzzle

1) The sum of the measures of the angles of a triangle is 180°.

2) If a triangle is equilateral, then it is equiangular.

3) If two sides of a triangle are congruent, then the angles opposite these sides are congruent.

4) The measure of an inscribed angle is one-half the measure of its intercepted arc.

5) Every angle inscribed in a semicircle is a right angle.

6) The measure of an arc is the number of degrees in the central angle that intercepts the arc.

AMAYTC Intellectual Development Standard
I-3: Reasoning

Perpendicular Bisectors of a Triangle

For this activity, the following equipment will be needed: a 5 or 6 inch square of unlined, lightweight, white paper like tracing paper and a ruler.
Do this activity in groups of three. Assign each team member either an acute, right or obtuse triangle.

- Draw your assigned triangle on the paper. Make it large and be sure to use a straightedge when drawing.
- Select one of the sides of the triangle. Fold the paper so the two endpoints of that segment meet. Fold it as accurately as possible. Crease the paper. The crease is a perpendicular bisector of that side of the triangle.
- Reopen the paper and fold the other two perpendicular bisectors in the same manner.

Write a conjecture about the perpendicular bisectors of any triangle. Look up the definition of a circumcenter.
Extension: Measure the distances from the point of concurrency to the vertices of the triangle. Make a conjecture about these distances.

AMAYTC Intellectual Development Standard
I-3: Reasoning

Triangle Inequality Theorem

For this activity use the set of manipulatives your instructor provides which might be strips of paper, straws or pipe cleaners that are 2, 3, 4, 5, 6, 8, and 9 inches long. Each group will have one set.

- Is it possible to build a triangle from the 2, 5 and 6 inch pieces? Keep a record of the lengths of the sides used and whether a triangle was possible.
- Do the same thing with lengths 2, 5 and 8 inches pieces and record the results.
- Repeat the experiment with 3, 6 and 8 inches pieces and record the results.
- Repeat again with lengths 4, 5 and 9 inches pieces. Record the results and answer the following questions.

a) In your own words, make a conjecture about the results of the experiment. Can any lengths be used to form a triangle? If not, which ones will work and explain why.

b) Given two sides of a triangle, does there appear to be a minimum and maximum for the length of the third side? If a triangle had sides of 10 inches and 14 inches, what would be the minimum and maximum length of the third side?

c) Describe, **in general**, how to find the minimum and maximum length of a third side of a triangle given the lengths of the other two sides.

AMATYC Content Standard C-3: Geometry

Sum of the Angles of Any Polygon

A pair of scissors, a protractor and a straightedge will be needed. The students know the sum of the measures of the angles of a triangle is 180°. Each group is asked to "count off" starting with 4, 5, 6 …10. (Note: there will be 7 groups of students in the classroom.)

- Draw your assigned polygon using a straightedge, with the number of sides given to your group by the "count off". Make the polygon large so it's easy to work with. Number the angles 1, 2, 3 and so on. Put the numbers *very* close to the vertex of the angle.
- With a protractor measure all the angles in the polygon. What do you observe about the sum of the angles of your polygon?
- All groups put your data in a chart for the entire class to see and make a conjecture about a formula for the sum of the measures of any polygon with n sides.

Number of sides	3	4	5	6	7	8	9	10	n
Sum of angles									

AMATYC Content Standard C-3: Geometry

Drawing Three Dimensional Figures

It can be fun and challenging to draw 3-D figures. This skill is very important in computer graphics. It may be helpful to do the drawing on graph paper.

To draw a rectangular solid from a front view:

1. Draw a parallelogram as the top of the prism.
2. Draw three vertical line segments to represent the three edges that are visible from the front view perspective.
3. Draw the bottom face. It is congruent to the top face. In mathematics we use dotted lines to show edges that are not visible. This produces the illusion of three dimensions.
4. Connect the vertices that are hidden using dotted lines.

To draw a cylinder from a front view:

1. Draw a circle for the top. Draw it slightly oval in shape.
2. Draw two vertical line segments to form the sides of the cylinder.
3. Draw a circle for the base that is congruent to the top. Use a dotted line for the "back" of the circle because it is hidden.

Here’s your chance to learn to sketch a square pyramid from a frontal view.

1. Draw a parallelogram for the base. This may seem strange for a square pyramid but the drawing is from a front view perspective.
2. Light sketch in the two diagonals to locate the center of the base.
3. Draw a vertical line from the center to the desired vertex of the pyramid.
4. Join the vertex with the “corners” of the base.
5. Make the hidden edges dotted lines.
6. Erase construction lines.

To draw a sphere from a front view:

1. Draw a circle and place a dot in the center.
2. Draw an oval around the middle of the circle touching the sides but not through the center.
3. Make the “back” part of the oval dotted since it is hidden.

AMATYC Content Standard C-3: Geometry

Make a Tetrahedron from an Envelope

For this activity a small envelope measuring approximately $3\frac{5}{8}$ inches by $6\frac{1}{2}$ inches, a straightedge and a pair of scissors will be needed.

- Seal the envelope closed.
- Fold the envelope in half the short way. Make a **very sharp** crease. Open the envelope flat.
- Draw the two diagonals of the envelope using a straightedge.
- Fold the envelope along the diagonals, one at a time, creasing the folds **sharply**. Open the envelope flat.
- With a scissors, cut out one of the obtuse triangles. Remove this piece.
- Open the envelope and fold it along the first fold made.
- Tuck one “side” of the envelope into the other “side”.

You now have a tetrahedron. There are many questions you can ask the students.

1. How many faces are there?
2. How many edges are there?
3. How many vertices are there?
4. Do the faces appear congruent?
5. What is the area of each face?
6. What is the surface area of the tetrahedron?

Resources:

1. The activity was adapted from http://tremor.nmt.edu./tetra.html.
   You will find very nice diagrams for folding and cutting at this site.
2. NCTM Student Math Notes, January 2001, Tetralope by David Masunaga,
   Available on-line if you are a member of NCTM at http://my.nctm.org/eresources/article_summary.asp?URI=SMN2001-01-1a
   David Masunaga has a complete handout that can be duplicated and used in the classroom with a wonderful extension to this activity.

AMATYC Pedagogy Standard P-1: Teaching with Technology

Teaching with Technology

This activity is a demonstration to show why calculators say "error" when asked to find tan 90°. It is done by using a spreadsheet, the table function on a graphing calculator or by completing this table using a calculator:

x	87°	88°	89°	89.5°	89.75°	89.8°	89.9°	90°
tan x								

Solution:

x	87°	88°	89°	89.5°	89.75°	89.8°	89.9°	90
tan x	19.1	28.6	57.3	114.6	229.2	286.5	572.9	

“Standards of Pedagogy recommend the use of instructional strategies that provide for student activity and interaction and for student-constructed knowledge.”

“… students learn more efficiently when their teachers structure new information for them and help them to relate it to what they already know.”

Source: <u>Crossroads in Mathematics</u>

"Geometric/visual thinking is an important strand in many areas of mathematics, including contemporary applied mathematics – problems rooted in geometric questions are central to the development of new technologies such as medical imaging and robotics. However, there has been a dramatic decline in undergraduate offerings in geometry. Further, solid geometry has long been gone from the high school curriculum. "

Source: Undergraduate Program and Courses in Mathematical Sciences Curriculum Guide, draft 4.2, Mar 25, 2003.