Investigation: The Order of Operations

The operations of arithmetic are addition, subtraction, multiplication, and division. When an expression has more than one operation, we need rules that tell the order in which to complete the operations and that tell us the purpose of parentheses. These rules are called the **order of operations**.

Answer the question for each example.

$4 + 12 \div 4$ $= 4 + 3$ $= 7$	**1)** Which operation is done first: addition or division?
$4 + 2 \cdot 3$ $= 4 + 6$ $= 10$	**2)** Which operation is done first: addition or multiplication?
$10 - 8 \div 2$ $= 10 - 4$ $= 6$	**3)** Which operation is done first: subtraction or division?
$18 - 6 \cdot 2$ $= 18 - 12$ $= 6$	**4)** Which operation is done first: subtraction or multiplication?

5) Describe what you learned about the order of operations in the above examples.

$7 \cdot 3^2$ $= 7 \cdot 9$ $= 63$	**6)** Which operation is done first: multiplication or evaluation of exponents?
$4^2 \div 2$ $= 16 \div 2$ $= 8$	**7)** Which operation is done first: division or evaluation of exponents?

8) Describe what you learned about the order of operations in the last two examples.

$12 \div 2 \cdot 4$ $= 6 \cdot 4$ $= 24$	**9)** In this expression, which operation is done first: multiplication or division?
$12 \cdot 2 \div 4$ $= 24 \div 4$ $= 6$	**10)** In this expression, which operation is done first: multiplication or division?
$10 \cdot 8 \div 2$ $= 80 \div 2$ $= 40$	**11)** In this expression, which operation is done first: multiplication or division?

12) If an expression only has multiplication and division, describe how to determine which operation to do first.

$12 + 2 - 4$ $= 14 - 4$ $= 10$	**13)** Which operation is done first: addition or subtraction?
$12 - 2 + 4$ $= 10 + 4$ $= 14$	**14)** Which operation is done first: addition or subtraction?
$6 - 2 + 3 - 4 + 12$ $= 4 + 3 - 4 + 12$ $= 7 - 4 + 12$ $= 3 + 12$ $= 15$	**15)** Which operation is done first: addition or subtraction?

16) If an expression only has addition and subtraction, describe how to determine which operation to do first.

17) The order of operations are rules that tell us the order in which to evaluate exponents, division, multiplication, addition, and subtraction. Based on all of the previous examples, describe these rules.

Evaluate each expression. Follow the order of operations. Show all intermediate steps.

18) $6 + 12 \div 2^2$ **19)** $4 \cdot 3 + 8 \div 2^2$ **20)** $2^3 + 4 \cdot 3 - 8 \div 2$

Answers: 18) 9; 19) 14; 20) 16.

Sometimes an expression includes parentheses ( ) or brackets [ ]. Always evaluate expressions that are inside parentheses or brackets first, following the order of operations. Once everything inside the parentheses is evaluated, the parentheses are eliminated. If there is more than one set of parentheses, do the evaluating in the innermost set first (work from the inside out).

Evaluate. Follow the order of operations. Show all intermediate steps.

21) $(12 + 4) \div 2 + 9$ **22)** $25 - (6 + 2) \cdot 3$ **23)** $25 - 6 + 2 \cdot 3$

24) $[(25 - 6) + 2] \cdot 3$ **25)** $(12 + 35 + 7) \div 3$ **26)** $15 + 20 \div 2$

27) $(10 + 20) \div 2$ **28)** $15 + (20 \div 2)$ **29)** $30 \div 6 + 4 \cdot 3$

30) $30 \div (6 + 4) \cdot 3$ **31)** $[(30 \div 6) + 4] \cdot 3$ **32)** $3^2 - 2 + 3 \cdot 8$

33) $2 \cdot (6 + 17 + 8)$ **34)** $48 \div 2^4 \cdot 6 \cdot 1$ **35)** $24 - 12 \div 3 + 1$

36) $12 \div 0$ **37)** $0 \div 12$

Answers: 21) 17; 22) 1; 23) 25; 24) 63; 25) 18; 26) 25; 27) 15; 28) 25; 29) 17; 30) 9; 31) 27; 32) 31; 33) 62; 34) 18; 35) 21; 36) undefined; 37) 0.