WORKSHEET 1: Introduction to a common set.

Each group has been supplied with a **deck of cards** which we will hereafter refer to as the **UNIVERSAL SET** for these activities, and will nickname “**D**”.

1) The *n-value* for a set is the number of things in it. Those things in a set are called its *elements*. The statement “*n(S)*” is shorthand for “the number of elements in set S”. For your set (aka deck) of cards, give n(D).

2) How are the elements of your D alike one another?
Discuss, and list conclusions here.

3) How are the elements of your D different from one another?
Discuss, and list conclusions here.

4) Give a name for each element of D. Make sure there can be no confusion about which card has which name.

5) Did you develop a guideline or rule or pattern to help you come up with the many names needed to give one to each of the individuals in the set D?
If so, describe it here.

WORKSHEET 2: Introduction to common notations.

Subscripting and Superscripting for humans, Initialing for word processors.

1) Give several shorthand methods to write "the ace of spades".

2) Since you will be exchanging work with other students today, it may reduce confusion and transition time if we all use the same notation to write about D and its elements, for today. Come to a consensus within your group as too which one of these notations we should use for a classroom *standard* today.

3) Class consensus.

Defining Sets with "Element-wise" and "Set-builder" notations

Use the element notation decided above to enable a complete set-notation description of D as a mathematical entity.

1) As a complete, element-wise list:

2) As a list aided by the *ellipsis*:

3) Using set-builder notation:

This is a format using symbols that includes both a description of the elements to be drawn from for consideration, and a description of the properties any elements must possess to be included in the set being identified. Set-builder notation is modeled by the following:

{<*domain* of elements> | <*condition* restricting which will be included>}.

Example: {whole numbers, x | 10 < x < 50} would be read "the set that includes all of the whole numbers which satisfy the conditions of being both greater than 10 AND less than 50". It is also often read as "the set of whole numbers x such that x is both greater than 10 and less than 50."

D = { | }

4) Class comparisons.

You should be able to look at any of the above 3 statements and know exactly whether you are working with a standard deck, or a pinochle deck, or a short deck. You should also know whether the universal set you are working with should include, or not include, any jokers or other cards which come with the purchase of a deck of cards!

WORKSHEET 3: Partitions of the set D.

"A **partition** Π of a set S is any *collection* of *subclasses* A, B, C, ... of S such that each element of S belongs to one and only one of the subclasses of the collection."

pg. 213, Groups, Georges Papy

Each group has been supplied with a bunch (deck) of cards which we refer to as the set D. To partition D, simply sort its elements into a bunch of separate piles. Each pile is a *subclass* of D, and the collection of piles is the "partition Π." We will do this in such a way that we can use our notations for elements and sets to document our work, and make it reproducible.

1) Take the set D, the universal set, and randomly sort it into some smaller piles (which we'll call subclasses, or subsets, of D).
2) Give a name to each pile you've made.

3) Use these names to describe your collection of piles as a set.
(ie. use Π = {list of names of piles, separated by commas}.)
Notice that here you are making a set which has sets for its elements!

4) Give the number of piles you've made: n(Π) =

5) Give the number of cards in each of the elements of your Π.
n( ) = n( ) =
etc.

6) What is the sum of the numbers given in part 5?

7) Make a list of the cards in each of the elements (piles) of your partition Π of D.

8) Give a "rule" using words which will enable another group to exactly duplicate the piles you've made (if possible, without having to look at the lists you made in part 5).

9) Write this rule down and exchange with another group. Duplicate their partition. When done, exchange your lists from part 7 to check your work. Discuss problems.

WORKSHEET 4: Partitions by nice relations among elements of the set D.

1) Take the set D, the universal set, and sort it into **equally-sized** piles.

2) Give a name to each pile you've made. Use these names to describe your collection of piles as a set.

$\Pi = \{$

3) Give: $n(\Pi) =$

4) Give the number of cards in each of the elements of your Π. Since they are all of the same size, we can write it as: $n(\quad) = n(\quad) = \ldots = n(\quad) =$

5) What is the product of the numbers given in parts 3 and 4?

6) Give a "rule" using words which will enable another group to exactly duplicate the partition you've made.

7) Write this rule down and exchange with another group. Duplicate their partition. When done, check each others' work for accuracy. Discuss problems.

8) When attempting to duplicate a partition that is an equivalence relation (like this worksheet?), which is easier to use: full lists of elements in the subclasses, or a rule?

When attempting to duplicate a partition that is a randomly assigned relation, (like worksheet 3?) which is easier to use: full lists of elements in the subclasses, or a rule?

"A relation x ~ y which is defined on all elements x, y of a set S, and for which it is true that:

1) x ~ x, for all x in S (Reflexive Property)
2) x ~ y iff y ~ x (Symmetric Property)
3) x ~ y and y ~ z together imply x ~ z (Transitive Property)

is called an **equivalence relation**, and ~ defines a partition of set S into **equivalence classes**."

pg. 213, Groups, Georges Papy

Example:

"Separate the deck into piles that each contain cards which have the same pip shape" illustrates an example of a relation between the elements of D. Any card has the same pip shape as itself (so satisfies the reflexive property). It is true that card 1 has the same pip shape as card 2, if and only if it is true that card 2 has the same pip shape as card 1 (so this relation satisfies the symmetric property). If card 1 has the same pip shape as card 2, and card 2 has the same pip shape as card 3, then we can conclude that card 1 has the same pip shape as card 3 (so "has the same pip shape as" is a relation that also satisfies the transitive property). So, we can conclude that "has the same pip shape as" is an equivalence relation on set D.

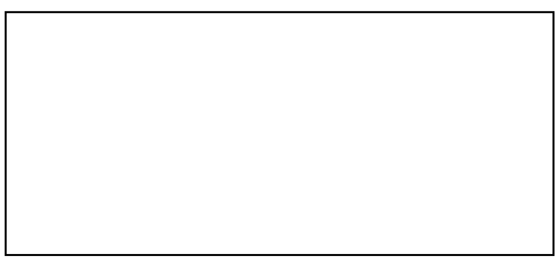
WORKSHEET 5: Partitions and Venn Diagrams

The most typical equivalence relations applied to D are generally referred to as:
"Suits" with 4 equally-sized equivalence classes, S = {diamonds, hearts, clubs, spades};
"Kinds" with 13 equally-sized equivalence classes, K = {aces, twos, 3's, ... , queens, kings};
"Colors" with 2 equally-sized equivalence classes, C = {reds, blacks}; and
"Types" with two equivalence classes of different sizes (number of people cards is 12, number of number-of-pips cards is 40), T = {picture of person, number of pips arranged}.

In these as in all partitions, the universal set is divided-up into non-overlapping classifications/subsets. There are no leftovers in this scheme--every element gets included under some description. The human race has had the notion that a picture can be divided up this way, simply by the drawing of lines on it. Thus, we use such pictures to illustrate the partitioning of sets. These pictures are often called "Venn diagrams". In a Venn Diagram, an area inside the picture's boundary line is identified with a classification, or subset, of the universal set.

Pretend that this rectangle has been divided up inside so that each card in the deck is assigned an equal portion of the entire area. A problem might be: decide which spot inside the box gets assigned to which card.

This problem is generally alleviated by designating certain areas to each subclass of a given partition. Then all of the elements of any subclass go into that class's designated area.



Universal Set, D

1) Draw a thick vertical line down the middle. Then label each side with a color.

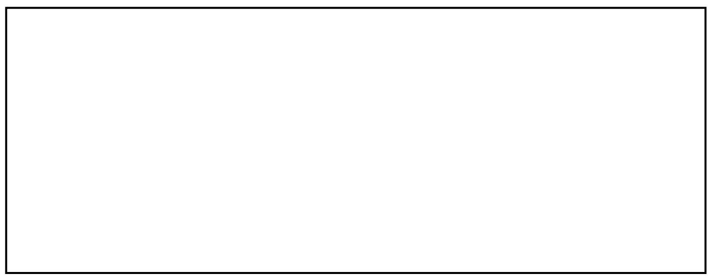
2) Draw a thin vertical line down the middle of each of these halves. Then label each side of each of these lines with an appropriate name for a suit.

3) If I tell you the suit of a card, can you tell me its color?
If I tell you the color of a card, can you tell me its suit?

4) If you were asked to list every element of D in the picture as divided and labeled above, which place in the picture would go with which card? Identify "home" for each of the 52 cards in the deck. How many belong in each of the four sections?

WORKSHEET 6: more Partitions and Venn Diagrams

Pretend that this rectangle has been divided up inside so that each card in the deck is assigned an equal portion of the entire area. Certain areas of the whole get designated to each subclass of a given partition. But we have seen already, that a set can be classified by several criteria--the several partitions that result may overlap each other.



Universal Set, D

1) Draw a thick vertical line down the middle. Label each side with a color for D.

2) Draw a thin horizontal line. Label each side of it with a “type” of card--either picture or numeric.

3) If I tell you the color of a card, can you tell me what type of card it is?
If I tell you the type of card it is, can you tell me its color?

4) If you were asked to list every element of D in the picture above:
which cards (elements of D) go in the upper left corner?
which cards (elements of D) go in the upper right corner?
which cards (elements of D) go in the lower left corner?
which cards (elements of D) go in the lower right corner?

5) Now draw a thin vertical line down the middle of the left half and one down the middle of the right side. Label each side of these lines with appropriate suits.

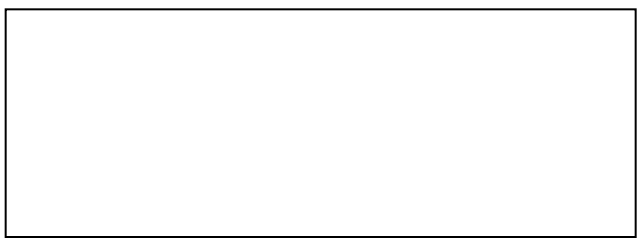
You now have “the universe” D cut into 8 parts. Use set-builder notation to describe which cards belong in each of the 8 subsets represented in the above picture.

7) Compare your picture and results with someone else’s. Are they identical?
Are they equivalent? Is there a difference between identical and equivalent?

WORKSHEET 7: special Partitions and circles in Venn Diagrams

A special kind of partition is very popular in our culture--that with exactly two subclasses. Thus elements can be sorted easily--they either go into one subclass or else they don't. Often, such partitions are defined as one set of elements that do exhibit a desired property, and everything else--(so the only other subclass is the set of elements which don't have the desired property).

When Venn diagrams are used to illustrate such partitions, circles are often used to represent the area of the rectangle set aside for elements with a given property. All elements that don't have the property are "inside the rectangle but outside the circle." Pretend that this rectangle has been divided up inside so that each card in the deck is assigned an equal portion of the entire area.



Universal Set, D

1) Draw a circle inside the rectangle. Then label each side of the circle with a color. (A circle has two sides--an in-side and an out-side. Inside is for the have's.)

2) Draw another circle to represent the two types (picture and numeric). Label each side of it with a type of card--either picture or numeric.

a) What cards need to be inside both of these circles? How many of them are there?

b) What cards are represented by the area inside the rectangle, but not inside either of the circles? How many of them are there?

c) What cards are represented by the area inside the first circle but outside of the second circle? How many of them are there?

d) What cards are represented by the area inside the second circle but outside of the first circle? How many of them are there?

3) Make a rectangle for D and draw circles to represent piles from the partition by suits. The four circles need not overlap, since they have no cards in common. What cards are represented by the area outside of the circles but inside of the rectangle?

4) Now make a circle for "red cards". Describe each "area" within the rectangle.

WORKSHEET 8: Functions on the set D.

"A **relation** r is defined as a set of pairs; its domain is the set of origins of its pairs and its image is the set of their endpoints."
"A **function** f : A ⇨ B (or, map f of A into B) is a relation of A to B such that every element of A is the origin of one and only one pair of f." pg. 211, <u>Groups</u>, Georges Papy

(A *function* is defined to be a set of ordered pairs, typically denoted (x, y). The x stands for any element from the domain set, the y stands for the particular element from the image set which is to be associated with the given x, via the relationship described by the particular function f. A partition is but a special kind of function. Using function notation, the pairs of the function called f are often written in the form (x, f(x))--thus specifically identifying the restrictions on possible y-values.

Frequently, the definition of f is given to you as a general rule, which states the relationship between each domain element, x, and its image value, y. To "evaluate a function" you must take each element of the domain set, examine it relative to the relationship described in the definition of f, and find/compute/determine the associated y-value/function value/image element.)

1) Choose one of the four equivalence relations for D, described earlier in this activity, to work with for this problem. Partition your domain (deck) accordingly.

2) Give each of your piles a one-letter name, and use these names to describe your collection of piles as a set. Use _ = {list of names of piles, separated by commas}.)
Notice that here you are making a set which has sets for its elements! This set lists all the possible places a card can be assigned to when making the above partition, thus is called an *image set*.)

I = {

3) Write a statement of how you decided which pile to put each card into.
This activity is also called "defining a function with domain D". It is like filling in what f(x) = .

4) Fill-in the given display for ordered pairs; first the name of a card (element of D) goes on the left, then the appropriate destination pile's name (element of I) on the right.
This activity is also called "evaluating the function you defined in part 3". It is like a function table or t-chart.

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(,)(,)(,)(,)(,)(,)(,)(,)(,)(,)(,)(,)(,)(,)

5) Does this list qualify as a function? Why, or why not?