DEVELOPING A MODEL OF ATMOSPHERIC PRESSURE ON MARS:
Connecting Concepts in Physics and Mathematics

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INTRODUCTION

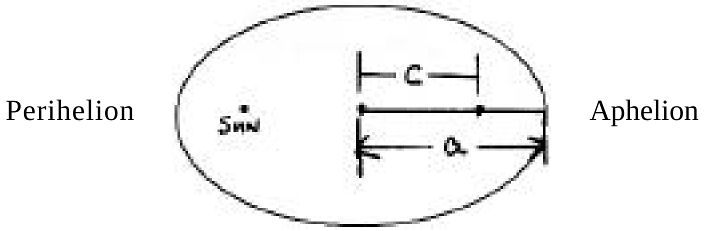
Since 1964, over 7,000 engineering and science educators from U.S. colleges and universities have participated in the NASA-ASEE (American Society for Engineering Education) Summer Faculty Fellowship Program in Aeronautics and Space Research. (For information and applications, contact www.asee.org).

The speaker was a fellow in the summer of 1998 at the NASA Goddard Institute for Space Studies (GISS). Her research goal was to develop a model of atmospheric pressure on the surface of Mars as a function of position on the surface of the planet and time of year, and to use this model to determine an improved estimate of annual global mean pressure.

Unlike Earth, atmospheric pressure on Mars varies greatly with the seasons. Data collected by the two Viking spacecraft from 1976 to 1982 has been analyzed and shows variation of about 20% annually. This is thought to be caused by exchange of CO_2 between the atmosphere and the planet's polar caps, as they sublime and condense. To better understand these variations, a review of basic laws of astronomy and meteorology follows.

KEPLER'S LAWS

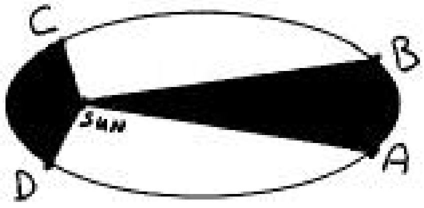
Kepler's First Law states that the orbit of each planet is an ellipse with the Sun at one focus.



Recall that the eccentricity of an ellipse is given by e = c/a where c = distance from center to focus and a = length of semi-major axis. The eccentricity of Mars' orbit is .09, as compared with .02 for Earth.

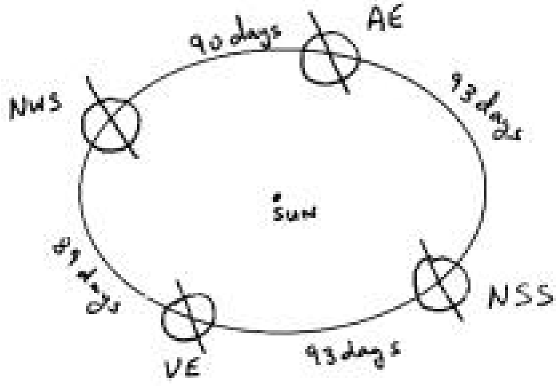
Kepler's Second Law states that a straight line joining the Sun and a planet (i.e., a radius vector of the ellipse) sweeps out equal areas in equal intervals of time.

As a consequence of Kepler's Second Law, when the planet is farther from the sun (as in AB), it covers a shorter distance and therefore travels more slowly. When the planet is closer to the sun (as in CD), it travels more rapidly.



SEASONS

The inclination of Earth’s axis (23.44°) is responsible for seasonal changes in lengths of daylight and darkness.



VE ≡ Vernal Equinox
NSS ≡ Northern Summer Solstice
AE ≡ Autumn Equinox
NWS ≡ Northern Winter Solstice

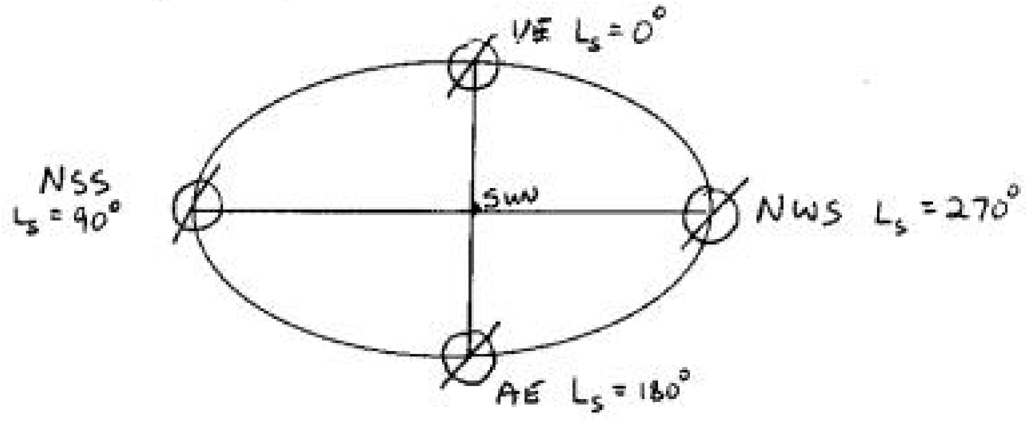
In the Northern Hemisphere, spring and summer are slightly longer than autumn and winter. This is because Earth is near aphelion in summer and is moving more slowly.

Summers in the Southern Hemisphere are slightly shorter but hotter (since Earth is near perihelion), and winters are longer but colder, than in the Northern Hemisphere.

SEASONS ON MARS

The inclination of Mars’ axis, 25.19°, is very similar to Earth’s, and so the solstices and equinoxes are defined in the same manner. However, due to the much greater eccentricity of Mars’ orbit, a seasonal index called the areocentric longitude of the sun, Ls, is used:

Ls = 0°	Vernal Equinox
Ls = 90°	(Northern) Summer Solstice
Ls = 180°	Autumn Equinox
Ls = 270°	(Northern) Winter Solstice



The following chart shows the lengths of the seasons on Mars in terms of Mars solar days, or sols. (One sol is about 24h 39m).

	Sols	Season (Northern Hemisphere)
0° ≤ Ls ≤ 90°	195	Spring
90° ≤ Ls ≤ 180°	178	Summer
180° ≤ Ls ≤ 270°	142	Autumn
270° ≤ Ls ≤ 360°	154	Winter

Note that Mars reaches perihelion at Ls = 251°. On Mars, summers in the Southern Hemisphere are considerably shorter but hotter, and winters are longer but colder, than in the Northern Hemisphere. As a result, the south polar cap of Mars frequently disappears during the summer, but the north polar cap does not. The polar caps are thought to be composed primarily of solid CO_2, and partly of water ice.

VIKING MISSIONS

Two identical spacecraft, each consisting of an orbiter and a lander, were launched in 1975. Viking Lander 1 (VL1) landed on 20 July 1976 at 22.48° N latitude, 47.8° longitude, and transmitted until 13 Nov 1982. Viking Lander 2 (VL2) landed on 03 Sep 1976 at 47.97° N latitude, 225.7° longitude, and transmitted until 11 Apr 1980.

Viking gave us data for the first weather report from the surface of another planet:

Light winds from the east in the late afternoon, changing to light winds from the southwest after midnight. Maximum winds were 15 mph. Temperature ranged from -122°F just after dawn to -22 °F. Pressure steady at 7.7 mb.

Note that on earth, sea level pressure is 1 atm = 1013.25 mb = 760 mm Hg = 29.9213 in Hg. Pressure on Mars is very low compared to Earth for two reasons: the density of Martian atmosphere is significantly less than Earth's, and surface gravity on Mars is only 38% of that on Earth.

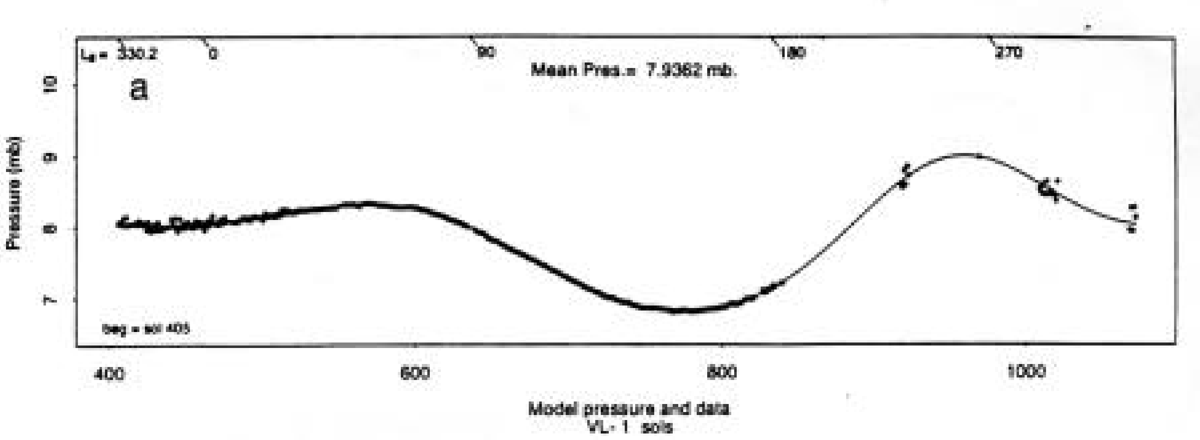
Based on data from Viking Landers 1 and 2 from 1976 to 1979, a model of atmospheric pressure at those two points on the planet was developed using weighted least squares [Tillman et al., 1993]. The equation below is from one year of data from VL1, starting with Sol 405. (Sol 0 is taken to be 20 JUL 1976, the date that VL1 landed on Mars).

$$P_{sol} = 7.936 + \sum_{i=1}^{5} [P_i \sin(2\pi i\,(S-405)/668.59692 + (\Phi_i(2\pi/360)))] \qquad (1)$$

where

S = sol number at which pressure is to be calculated
The P_i values (i = 1 to 5) are .661, .574, .113, .065, .014
The Φ_i values (i = 1 to 5) are 91.96, -129.23, -66.20, -4.04, 25.6
668.59692 is the number of sols (solar days) in the Martian tropical year (equinox to equinox).
7.936 is the mean pressure in mb for the year at VL1.

The maximum pressure at VL1 was 8.999 mb, occurring at Ls 260°. Note that this is just before the summer solstice in the southern hemisphere, and just after perihelion. The minimum was 6.813 mb, occurring at Ls 148°.



THE HYDROSTATIC APPROXIMATION

Results from VL1 can be extrapolated to other locations by using the hydrostatic approximation:

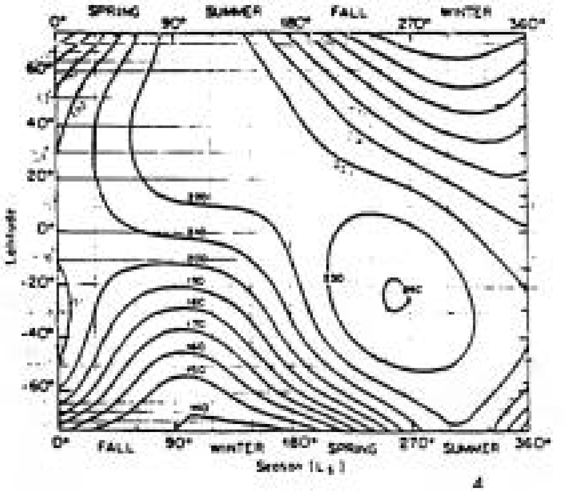
$$\ln(p_2/p_1) = -(mg/RT)(z_2-z_1) \qquad (2)$$

where p_2 = pressure at point z_2,
p_1 = pressure at point z_1
g = acceleration of gravity,
R = universal gas constant
T = mean temperature between z_1 and z_2
m = mean molecular weight between z_1 and z_2

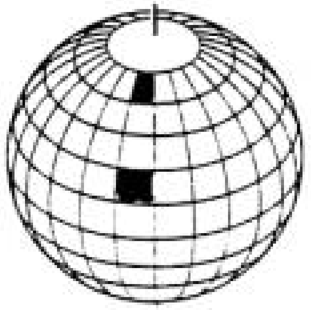
Solving the equation above for p_2 gives $p_2 = p_1\, e^{-(mg/RT)(z2-z1)}$ (3)

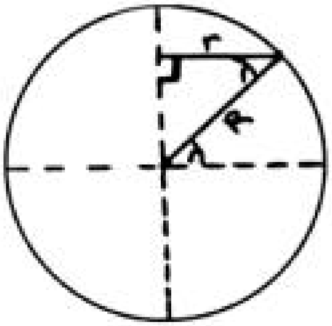
DEVELOPING THE MODEL

To develop a model of atmospheric pressure at any point on the planet, I incorporated equation (1) for p_1 into (3), using Mathematica on a Macintosh. For Mars, $g = 3.73\ m/s^2$ and R/m is $192\ J\ K^{-1}\ kg^{-1}$. The chart below [Kahn, 1983] was used for temperature. Input data for the GISS Mars General Circulation Model (GCM) was used for heights.



The Mars GCM has a resolution of 8° latitude by 10° longitude. Gridboxes are not of equal area.





$r = R \cos \lambda$

The distance corresponding to one degree of longitude varies with latitude. A parallel of latitude at latitude λ has a circumference equal to $2\pi R \cos \lambda$ where R = radius of Mars

To calculate the average annual pressure, I proceeded as follows;

- For each latitude, sum the pressures over the 36 longitudinal divisions, and divide by 36;
- Multiply by the cosine of the latitude and sum over the 24 latitudinal gridboxes;
- Divide by the sum of the cosines of the latitude of the gridbox midpoints;
- Sum over 668 sols (one Martian year), and divide by 668.

This gave an annual pressure of 5.90849 mb. Next, I used an interpolating function for the height, and integrated rather than summed.

CONCLUSION

The hydrostatic approximation applied to modeled pressure from Viking data was used to develop of a model of atmospheric pressure on the surface of Mars. Preliminary findings give a mean annual global pressure of 5.90849 mb.

Future plans include extending the model to predict pressure as a function of time of day.

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APPENDIX

	EARTH	MARS
Equatorial Radius	6378 km	3397 km
Mean Solar Distance (Semi-major axis)	1.00000 A.U. = 1.49598 x 10^8 km	1.52366 A.U. = 2.27936 x 10^8 km
Orbital Eccentricity	0.0167	0.0934
Surface Temperature	287°K	214°K
Surface Pressure	1013 mb	5.6 mb
Mean gravity	9.8 m/s^2	3.7 m/s^2
Tropical Year (equinox to equinox)	365.2422 days	686.9725 days = 668.5921 sols
Sidereal Day	23h 56m 04.09054s	24h 37m 22.66s
Solar Day	24h 00m 00s = 1 day	24h 39m 35.245s = 1 sol
Obliquity of Equator to Orbit	23°.44	25°.19

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