

Integrating Technique with Content

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[This text is a close approximation to the talk I gave at AMATYC Conference in Pittsburgh, 1999. The spoken material, here reconstructed from notes and memory, is given in written form interwoven with the overheads used, keeping the same sequence used in the actual talk, keeping the conversational tone.

Most of the pages in this document are copies of overheads, so include very little writing.

Brackets indicate notes to the reader, not from the actual talk.]

I made two proposals to AMATYC to speak here today. Both covered the same topics; one was for a two-hour discussion format and the other for a one-hour presentational format. They invited me to do a one-hour presentation of my work, so this hour will be mostly me talking. Hopefully time will allow some open discussion at the end, but I'm afraid I doubt it.

Note that everything that will go on the overhead, which will be a comprehensive outline of everything I'm going to say today, is included in the handout in a shrunken form. You are free to give your attention to the content of the talk, rather than spending your energy trying to document it!

This talk will be a report from a teacher. While rooted in formal research it is given merely as the observations and reactions from a normal classroom-- which, it seems to me, is never as controlled a setting as formal researchers desire. My primary references will be from ancient history (the 1940's) and from my own experience.

The ideas behind this talk were planted and nurtured while I was a college student in Santa Barbara and Missoula. I feel I benefited greatly from having matriculated with several, in particular, progressive mathematicians who just happened also to be interested in the teaching and learning of mathematics. (Paul Kelly at UCSB; Merle Manis, Johnny Lott and others at UM).

Here's a story to sort of set the tone. It comes from my senior year as a mathematics student at the University of Montana in Missoula. It could be rather long and complex, as many real math problems tend to be, but I'll try to cut to the chase. There I sat in the office with the department chair, telling him of our dilemma.

"Dr. Ballard gave us two weeks to do 6 of 10 problems as the take-home portion of the final exam in our abstract algebra class. Well, as you might guess, the problems were not trivial. After a week we started talking to each other--no one had more than one or two solved, and we all began to panic. So we decided

to get together and hammer some out. All but two of the class spent nearly an entire day working together, and we actually were able to get, we think, seven good proofs. But now the panic has shifted. Did we just cheat? Did our panic to pass a course in our senior year just cause us all to be expelled?"

"Well, I think you should go talk to Dr. Ballard about it. He doesn't bite you know."

This was reassuring to hear. Experience had taught us that he was a very difficult and very demanding instructor. We'd never seen him bite, but certainly didn't doubt he could. So we took the story down to his office. He listened carefully with an unreadable expression, and followed up with some cross-examination to clarify the details. Then, with his face kind of twisted up, somewhere between a grimace and a grin, he gave his verdict, slowly and solemnly.

"I guess if you all felt compelled to devote the time and energy to share your work in order to complete this test, then I have no choice but to give you all extra credit for your efforts. Don't forget, in your final write-ups, to give credit where credit is due."

That day I learned more about mathematics and education than in my entire college career. From the mouth of the most very traditional, old-school professor came clear support for what was then considered radical learning techniques--cooperation and content writing. I have never forgotten the power of that lesson, nor the power of our cooperative efforts in solving some very difficult problems in mathematics.

This set the tone for my secondary teaching, later my graduate studies, and later yet my college teaching. As a grad student I delved into research on "cooperative learning to enable student work with non-trivial integrated mathematics curricula." Luckily, my course coordinators at UM were highly supportive of bringing these ideas into my classrooms, so I did--for several years.

Now in my third year at the University of Alaska Southeast-Ketchikan Campus (which was originally just Ketchikan Community College) I have continued to implement these ideas. I have tried things and assessed results and adjusted and honed to get the results I'll talk about today.

My student's at UAS-K are primarily middle-aged working people with families at home. Many are old enough to have successfully avoided any study of mathematics in high school. Most know their addition and multiplication tables, though most also don't know when or how to use them. Living in remote

Alaska they have come to like satellite tv and nearly-fresh vegetables, and my students are those who have decided to finally learn something about the mathematics and technology which have enabled such luxuries.

Many believe the common knowledge, as Tom Lehrer put it in 1961, that “in the new approach, as you know, the important thing is to understand what you’re doing rather than to get the right answer.” (from the song “New Math”) My students learn quickly, however, that in the new New Math, they must get the right answer as well as understand how.

They are neither traditional students, nor a random sample from a normal population. There are not enough students here to support a control group. Thus, again, my results do not qualify as official research. They are, however, important enough in the life of normal classroom teaching in the two-year college setting, to warrant this discussion. At least they are to me [written and verbal responses from attendees at the talk suggested that these results are worthy of consideration by other teachers as well].

It is also worth noting that I am the only mathematics faculty at UAS-K. In any year I teach the entire spectrum of courses from pre-algebra to calculus, along with liberal arts mathematics courses. I am interested in results applicable and useful to all entry level college mathematics students, and will not focus on any one particular course.

So, enough with the background information. Let us move into the meat of the matter. [Overhead 1]

IRONY:

Mathematics developed in 1800's

is called "new math"

Teaching techniques developed in
1930's

are called "traditional"

Those techniques in mathematics education, now called traditional, were developed by the Behaviorist school of psychologists first for military personnel, then for the industrial style high schools that were sweeping our nation in the 1920's and '30's. Many mathematicians immediately recognized the severe flaws in the Behaviorist movement to dis-integrate mathematics. Opposition began immediately.

Several classic texts and treatises were published to point out the flaws of the Behaviorist assembly-line approach to mathematics. (Note that in today's world a "classic text" can be best defined as one where the title and author are familiar to nearly everybody, but which have been actually read by almost no one.)

Courant and Robbins reminded us of the fullness and richness of this majestic tool called mathematics. They noted that the six general attributes of What is Mathematics? were not well represented by the discrete model being promoted in the schools. George Polya then wrote volumes to help public school teachers in their efforts to teach mathematics as an integrated and accessible field of study, to enable students to learn this problem-solving tool in all its power.

[Overhead 2]

in response to 1930's trend to separate
mathematics into discrete "lessons"

What is mathematics?

Courant (and Robbins), 1940

Mathematics includes all of the following:

Logic

Intuition

Analysis

Construction

Generality

Individuality

Polya's discovery methods integrate all
these "antithetical pairs"

(mathematical discovery vols. I & II, and how to solve
it)

But such efforts were to little avail. The (now called) traditional approach took firm hold of our schools by 1950. The effort to isolate the body of mathematics into discrete chunks continued. It removed from the teaching of mathematics fully half of the attributes suggested by Courant--intuition, individuality and construction were dropped from the mathematics curriculum.

This traditional approach is a favorite among publishers and schools--from primary through college--to this day. It works effectively to identify and "weed out" that 10% or so of our population which: live high in the normal curve for symbolic manipulation skills; thrive in well-defined work of an isolated and solitary nature. It doesn't address problem-solving of the human sense--found in the vagaries of life as social beasts--but rather focuses on exercises which are well-defined, have already been solved, and are so wrought with assumptions and simplifications that they barely qualify as "problems."

[overheads 3 and 4]

60 years of history have shown us that:

The Traditional 10%
talented in
logico/symbolic skills
(as assured by the normal curve)

--DO learn from the traditional
approach

to do easily-testable mathematics skills

(the left half of what Courant called Mathematics)

Logic, Analysis, Generality

--But few of these students learn

applications that are difficult to test

(the right half of what Courant called Mathematics)

Intuition, Construction, Individuality

--and this ignores the other 90%
of human beings

Too many “good students” opt out of
mathematics and science
as soon as the opportunity presents itself.
Why?

The Second Tier Sheila Tobias’ construct

The most significant contributing factors are the lack of:

- 1) opportunity to develop facility
with the language of the content
- 2) opportunity to develop facility
with the “language” of human interaction
- 3) shared success, and the strength of spirit it engenders.

Communication and Belongingness

By the late 1970's we had begun to see results from this traditional approach. In America (as in China), even among that elite (though shrinking) group who had a clue how to do mathematics, there were few that knew how to apply mathematics to the world around us. And as for the rest of the population, ... well who really cares, eh?

It turns out that leaders in business and industry and the military now care also about the mathematical skills of people who aren't in that elite 10% group. They have motivated much research and much comparative testing. A technologic society requires that everyone be facile with the underlying mathematical structures.

The TIMMS survey noted that American students, trained under the traditional Behaviorist approach, claimed to know math as well as anybody (have near the highest self-esteem in the world), and yet when asked to show their stuff ranked near the bottom of the pack. They may be "a mile wide" but make up for it by being simultaneously "an inch deep." Thus, they get stuck a lot when they try to float even mildly complex problems.

Meanwhile, the technologic revolution and information rage advance at unprecedented rates. The rapid progress of modern society is based in the use of mathematical techniques and models. We use mathematics more now than ever in our history. Many of my adult students are here to learn how to use mathematics in their lives and careers.

How is it used?

Technical (including numeric) communication

Measurement for material concerns

Prediction via models

the weather

consumer demands

military strategy

design of technologic devices

research in general

Organize observations

pattern synthesis:

tables and formulae

logistics

meeting demands/promises

optimized effort

design

scaled drawings, circuitry

documentation

We humans do like to **manipulate** things
in both work and play.

Given the logistics of living in our material world,

we **measure and tabulate**

to obtain **information**

about that which was manipulated.

and then we **manipulate**

this **information**

popularly through

well-defined instruction sets and algorithms

Meanwhile, history and experience have taught us

that diverse people with common goals-

-to 'figure out' the math, science, business and life

benefit greatly if they **Communicate**
and Cooperate

while manipulating and tabulating

the things and the information about the things

These are **mathematical**
concerns

which are not addressed in the traditional curriculum and delivery, yet are necessary skills for prospering citizens.

The essence of democracy,

free enterprise and
the workplaces of the world.

By 1990 even academia had acknowledged the call for reform from the business, industry and military leaders of our nation. It had begun (with more than a little pressure from the likes of NCTM) to identify important goals, objectives and techniques. The tremendous insights of Courant, Polya and Maslow (of the distant past) can be found alive in the roots of the modern constructivist movement.

So, we've now looked at what it is and how it's used. Do these square with how it's taught and to whom? I had successful careers in both industry and business before thinking about these questions and concerns. My experiences both outside the classroom and in it, and my coursework as a graduate student were crystallized by a formal research question.

That question was very formal in its given state. I include here the formal, a cynical, and a practical restatement which focus it on the needs of a teacher rather than merely researchers.

Question raised one day in grad school,
“research in mathematics education”

“We have all these “new” tools and approaches developed for our student’s benefit. Each requires the teacher to know it, each requires time in and out of the classroom to implement. There is very little spare time in current mathematics course curricula.

How can the modern pedagogy be used
in such a way to maximize the
mathematical focus of contact time?”

Or,

How can we teach to 21st Century
mathematics standards,
while working in 20th Century
(traditional)

environments?

Or, to this teacher's particular point

How can we adopt AMATYC standards
for curriculum and assessment "reform"
in the classroom,
while chain of command prefers
the traditional old status quo?

What is status quo? Here I refer to use of the mathematics class to identify, weed out, and optimize that small portion of the population who could learn mathematics essentially on their own. It includes a deep rooted academic prejudice against teaching mathematics as a useful tool.

Yet the status quo is now being challenged with:

new tools

new social demands

new techniques

new goals

new student populations

competition from commercial schools.

Usefulness (vs. intrinsic beauty?)

This research question concerns
usefulness

--of everything done in the classroom

While calls for reform focus on
usefulness

--of the mathematics we teach

(While the Greek tradition--
where useful mathematics is considered
“home economics”

and thus beneath the integrity of
“the educated”--
persists in many American colleges.)

How can all these new be squeezed into the traditional classroom setting? I suggest that “onto” the already overwhelming list of discrete topics isn’t working.

A mathematical answer comes from history. When there are more concerns, we have looked to more sophisticated notions and notations. The concept is called “density.” This suggests the need to integrate content with technique, education with society--and both of these integrations are antithetical to the traditional industrial model of public education in America today.

Structures of the so called traditional approach, in fact, satisfy the mathematical definition of “nowhere dense”. And what can be disparaging to curious students and interested teachers is that this lack of density has been by design!

DENSITY

In mathematical analysis:

every neighborhood of a given point
contains infinitely many other points.

Applied to teaching mathematics:

every interaction on a given topic
touches upon many other topics

Does not exist in the traditional
approach:

whereby it has been declared desirable
that any interaction on any given topic
can be completely isolated from concern
for any other topic

(so can be identified and assessed with a multiple choice question)

Eisenhower Grants: in Ketchikan, AK

“The Four C’s”

Content

Careers

Community

Connections

Integrated mathematics includes:

usefulness as well as

intrinsic beauty as well as

“intellectual maturity”

Now looking back at that research question which motivated the work and this talk, there was a vague reference made to modern pedagogy. The calls for reform have come from the private sector, business, industry, military, government, and academia. My concerns are focused on those that address the needs of my students at the community college in Ketchikan. These include:

“new” Pedagogy Standards

focused by Content

mathematics skills, mathematics concepts, mathematical
ability

Construction

deductive systems from axioms
algebra generalized from arithmetic
modeling and manipulatives
logistics and discrete systems

Communication

content area writing
content area talking
symbolic representation

Cooperation

attitude
activity

Community and Careers

applications of mathematical problem solving

workplace mathematics
democratic decision making

“new” Pedagogy Standards pg 2
focused by Content: mathematics skills, concepts, ability

Integration and Connections

un-separate math topics
un-separate math from life
models and prediction
self-assessment

Technology

computing tools
communication tools
workplace and life tools
modeling

The Syllabus as a Contract

outcome goals
obligations and expectations
measurable performance assessment
weighting the C's appropriately

Mathematics for All

not just the traditional 10%

General Education Requirement of all degrees at UAS

Meanwhile, our campus catalog also has something to say about reform. At UAS reform concerns have been delineated and focused--in the daily life of teachers and students--via "Student Outcome Goals".

“Student Outcome Goals

...Assignments and tasks will be embedded into the course objectives of many different courses at different level of the curricula to provide students the opportunity to learn and demonstrate mastery of these competencies:

Communication:

...write speak, read, and listen effectively for a variety of purposes...

Quantitative Skills:

...analytic and mathematical reasoning...and quantitative problems.

Information Literacy:

...identify...locate...analyze and evaluate...integrate and communicate...evaluate information (as well as) product and process.

Computer Usage:

...skills range from basic abilities...to telecommunications...and electronic mail technologies.

Professional Behavior:

...is expected of college students...including responsibility, good work habits, ethical decision making,...and successful human relations.

Critical Thinking:

...approach from multiple perspectives...compare and contrast ideas and models, and willingness to take intellectual risks.

Students' skills in these six areas will be assessed periodically during their studies at UAS.”

Per the criteria of Courant and Polya, all of these outcome goals are in the purview of mathematics and mathematics education. Per the traditional pedagogy, only two (quantitative and critical) apply to the mathematics curriculum. Per my customer base at the entry level at a community college, Courant has the right idea.

Back in Missoula an older Analyst sat in on a seminar regarding reform of mathematics curriculum. He quipped “It doesn’t sound like reform at all, just a return to what is really mathematics.” (To which he referred to the days before the 20 pound math texts.)

So we have all these new goals and techniques--be they reform or return. We have student outcome goals. We have entrenched academics afraid to alter their 30 year old lesson plans. And to severely complicate matters we have “Math for All” as a General Education Requirement for all degrees, not just for the traditional fields (science and engineering) that have needed only the traditional 10% of students who can learn mathematics from the traditional approach. What to do?

For entry level courses in college mathematics,
and for remedial mathematics courses which dominate
the schedule at my community's two year college
campus,

in the effort to reach towards modern standards
(business, industry, AMATYC, NCTM)

in the traditional college environment
(as teacher, low in hierarchy of policy making)

I have focused on 5 "measurable" techniques and skills:

Content Area Writing
Portfolio maintenance
Cooperative efforts
Self-assessment
Manipulation

related to 1) things and 2) measures of the things

Now we face the next important question. How do we make these concerns, which are so important to the learning of mathematics, important to our students? How do we give weight to concerns that are “non-traditional” in American pedagogy (ie. since 1930’s), which are those same “traditional concerns” of mathematics (ie. since 500 BC)?

The answer comes from an analogy, a generalization of a commoners truth.

Old adage in business, industry and government,
when dealing with politicians and administrators:

If you want to know how
they
really feel about a thing,

don't bother to ask them
how they feel about it,

ask them how they have

budgeted for it.

In my post-secondary school
experience,
as older students try to interpret
what is “really” important,

they look at the “budget”.

To the degree program:

what courses are required?

To the individual courses:

what is required to pass?

Many students seem to primarily
acknowledge as important
those things which

contribute to their grade.

Which can be mathematically
communicated through the
syllabus as a contract.

To address all of:

Professional Standards (AMATYC)

School Standards (catalog)

Societal Needs (careers and home)

Student humanness

Teacher time !!!!!!!!

Integrate goals and expectations
through:

--Choice of text and problems
assigned

--Syllabus as contract

well-defined grading rubric

“your lowest possible grade will come from this
computation, based on these assessments”

admission of subjective influences

“I reserve the right to ‘raise this grade, but not lower it’
based on my observations such as ...”

... improvement, cooperation, effort

For the purposes of this talk, I will give only a quick nod to the very important choice of text and assigned exercises and problems. Then we will move on to the meat of the matter--the syllabus. [In Pittsburgh I used only the top half of the next overhead, and then skipped to the four-part grading rubric and the syllabus. Since not under the same time constraints here, I have left them in.]

I here strongly state my preference for applications based texts, and applications based exercises. Such an approach motivates and emphasizes: the human uses and the human development of mathematics; content communication and vocabulary extension; applicability of mathematics as a tool for daily life in our society; concern for human preoccupation with manipulation and measurement; and the foundation in reality for "this (measure) as a function of that (measure)... as well as the meaning of division.

Psychology: goal v. obstacles to the goal

Applications based texts and curricula:

--focus on historically human objectives (integrate with life)

 motivated by growth of the individual to live well
 within civilization and society

--motivate content communication and vocabulary growth

--illustrate relations between things and measures

--supplementary skill and drill worksheets very available
 require little teacher time/energy to implement
 are naturally motivated by desire to learn human tools

 to solve human problems

Traditional texts and curricula

--focus on abstraction and isolation from humanity
 motivation is “intrinsic beauty” and/or compulsion

 ignored by many as a troublesome elitist diversion

--make human appreciations and applications of math
 (ie. “word problems”)

 seem like confounding and confounded diversions
 seem like irrelevant add-ons to work in abstraction

--supplementary application problems (becoming available)

require much teacher time and preparation
 require significant effort to have students take them seriously

Applications do motivate traditionally required topics:

Topic via daily Application

Square roots: statistics, standard deviation
 (media literacy)

Root functions, triangle trig
 rational functions (construction,
 navigation)

Linear functions telephone and power
 service
 (“choose us!”)
 manufacturing \$ concerns
 (fixed and variable
 costs)
 retail \$ concerns
 (salary and commission)

Proportions, ratios hourly wage, travel, rates

Exponentials

banking and credit
copy machine sizing

PS: With each use of technology, i explicitly
state “In the old days, to do this i would need:
...”

scratch paper, book of tables,
slide rule, adding machine,
more time, etc.

The syllabus I have found to be extremely useful to adult learners, especially that majority of adult learners who expect my college math classes to be a repeat of the equivalent high school math courses. It must be mentioned again that the majority of my students at UAS-K either deliberately (and successfully) avoided such study in high school, or else they failed to succeed in that traditional system.

The syllabus is my 'day one' attempt to reduce the negative psychology carried by these entry-level students, and simultaneously to give them a clear understanding of the expectations of college mathematics. It includes the weighting of the components of learning mathematics--both traditional and reform. It includes a general outline of the course, but does not give daily assignments.

(I prefer to give students the responsibility to keep up. If they miss a day of class, they must face the fact that it is their responsibility to discover what they missed. This helps emphasize the need for continued and on-going effort. They cannot fool themselves by believing they can put it off, and then just work through the "predetermined assignments" in a mad, sleepless week. Also, it gives me the freedom to adjust daily assignments to the interests and needs of the particular students in the class.)

I use this four-part grading rubric (aka "random variable") as the outline for all of my courses. Depending on the level of the course, the weights of the four parts vary.

Grading of these four areas of assessment is highly integrated to balance modern with traditional expectations (of both students and teacher) in my mathematics classes. For example: the low grades that often result from the tough/non-trivial weekly quizzes are generally balanced out by the high grades that result from my scheme for grading the weekly writing assignment. More about these to come.

Syllabus for Math107--College Algebra (4 credits) ...

Your grade for this class will be determined by your performance across the four modes of assessment described below:

Preparation and Participation

135 points _ 13%

Tests and Quizzes

625 points _ 59%

Email writing assignment

_ 140 points _ 13%

Portfolio

160 points _ 15%

total possible _ 1060 points for
100% of semester grade

Note: Each component of this grading rubric is worth at least one full letter grade.

Note: This grade does not include a component for “student’s innate abilities.” Only student performance, as described above, is considered in the final determination of the course grade.

Using this grading rubric, the numbers work out so that:

If you can't write and communicate, and you can't keep track and organize, but you can accurately do symbolic manipulation (with or without context), then you can get a "C" in my class. This acknowledges traditional standards.

If you can write and communicate technically, and you can organize diverse information, but you can't do symbolic manipulation well (with or without context) then you can get a "C" in my class. This acknowledges industry standards.

You must write and communicate, organize technical information, AND accurately perform symbolic manipulations in order to get an "A" in my class. This acknowledges modern professional and world standards (see AMATYC, NCTM, and the TIMMS results).

Once upon a time this was a one-page syllabus. It required thought and enabled flexible application to diverse students. But some very legalistic students didn't want to think and interpret, and wouldn't accept that learning mathematics could be a flexible and adaptive effort. These few students forced, through administrative/political pressures, an extensively defined description of the meanings of the words used. I attempted to deal with this administratively mandated dis-integration in a constructive manner.

As a result, my syllabi now range between eight and ten pages long. I spend the entire first day of class reviewing the syllabus and clarifying definitions and expectations. (Legalistic students with perfect attendance can no longer convince themselves that "sitting in the back reading romance novels" should be interpreted as "perfect participation.") The second day of class begins with a quiz on the syllabus.

I also assign a worksheet called "Algorithm to self grade" in about the third week of class; this gives a detailed step-by-step accounting of student performance based on the expectations given in the syllabus. When students ask me "What is my grade so far?" I respond by asking them to break out the worksheet, and offer to work through it with them. Knowing where they stand, in terms of their grade-to-date, thus becomes the students' responsibility. Part

of learning mathematics is the ability to use it to answer such questions for themselves. (And their portfolios were designed to include all of the information needed to compute their grades. More on portfolios soon.)

When preparing novice adult students to step up to the level of college mathematics, I agree with Glenda Lappan's President's Message from the NCTM News Bulletin, September 1999. "Just send them to us knowing how to study mathematics and how to be organized and with a willingness to try things in mathematics that require some effort." In an effort to give weight to these attributes, so that adult learners may come to see the value in them, I persist in including Preparation and Participation in my grading schemes, especially in remedial algebra classes.

Preparation for and Participation in

Assesses and acknowledges: Effort, Desire, Focus,
Attention

Motivates: Effective practices and cooperation

1/3 = Attendance. “But this is only a beginning ... “
Homework assigned daily at beginning of every
class.

1/3 = Completed daily assignments daily

Measured through: verbal questioning and paper checks
(for cooperation, professional behavior, demonstrated work)

“A meaningful question about an assigned problem is
as indicative of preparation as is a correct solution.”

1/3 = Contributed to daily classroom discussions and
activities

Measured through: 5 minutes of compare/contrast quiz
grading;

verbal questioning, in-class activities, note-
taking

(cooperation, communication, basis for self-comparison)

Per the Alaska Administrative Code *Handbook for Alaskan Educators*: “The accepted general standard for (college) student pre-class preparation is a minimum of two hours preparation for each hour of lecture. That many students may have full-time jobs or are involved in other activities is not a valid reason for reducing course expectations.”

Concentration, focus and the ability to follow detailed (well-defined) instructions are learned skills; we practice these in class.

I do a significant amount of questioning of students in class. This helps with guiding their proper interpretation of language and symbols. For example, I do not accept the common “x four” when they need to read and say “x to the fourth power.” I make them say things correctly and frequently, and this does help them learn to interpret symbols meaningfully.

Note that reviewing their quiz results is integrated into the Participation portion of their grades. Students spend about five minutes comparing results with each other. This answers over 90% of all questions concerning “what is correct” and also “how partial credit decisions” were made. This activity also gives them direct data for self assessment and comparison of their progress relative to others in the class. I do not answer any questions until they have discussed them among themselves.

During this time I post the summary statistics for the quiz. This reinforces prior stats lessons and gives additional opportunity to experience interpretation of statistics--in a very real and relevant setting.

This activity, which takes very little class time, enhances a cooperative attitude among students, content centered communications, and mathematical self-assessment. It integrates quantitative skills, information literacy, critical thinking and professional behavior.

The quizzes and tests themselves tend to focus on more traditional aspects of math education. I use these to assess manipulation skills in quantitative settings which require a degree of critical thinking. To a lesser degree they also assess student knowledge of the meanings of technical vocabulary and mathematical symbols. Quizzes and tests often emphasize those applications problems which were demonstrated in class; this encourages students to take good notes and to review them at least once per week.

Assess the student's ability to DO mathematics.

Weekly Quizzes

Provides the math anxious with test taking practice in low impact setting.

Regular feedback for self-assessment--both instructor & student

Address material that was assigned, or discussed in class, during the prior week only.

Encourage note taking and review of notes.

Closed book, focus on recent learning, few topics

Provide outlines for review for exams (see portfolio)

Teacher time per class: < 1 hour to assemble;
< 2 hours to grade

Class time: 15 minutes to give, < 10 minutes for review

Tests

Address synthesis of material, techniques over several weeks.

Open portfolio

Encourages students to keep a good portfolio.

Enables non-trivial test questions.

Assess ability to find and use information (information literacy)

in mathematical problem-solving processes.

(like at home and on the job)

Participation, preparation, quizzes and tests together account for seventy-some percent of the grade. “Ace all of these assessments and ignore all the rest, you’ve got a C and are prepared for the next level.” Some students do. Others believe I have some ulterior motive in having them do the next two tasks. I do, of course. (shh it’s called useful learning)

Weekly E-mail journal / essay

Assesses “UNDERSTANDING” of mathematical topics,
feedback weekly:

often uncovers subtle misunderstandings,
improves technical work

Communications Skills:

requires a main topic,
requires a vocabulary word used in context
provides non-intimidating practice with technical writing

Critical Thinking and Creativity:

choose your own topic (of interest to you) each week

Computer Usage:

email accounts are provided by UAS upon registration
practice with keyboarding
provides non-intimidating interaction with computers
provides insight on limitations/capabilities of computers

Professional Concerns:

technical communications
deadline: once per week, no make-ups

In the grade, counts as much as the weekly quiz!

Takes almost no class time.
(Teacher time: _ 5 minutes each student, per week
respond in CAP LOCK mode)

This assignment gives awesomely powerful feedback.
It is worth every extra moment.

Graded (usually) as “all or nothing”:

Balances poor quiz performances: enables the 2nd Tier
Reduces intimidation factor: enables risk taking
I’m not their English teacher

*

But a great Concern: How to balance...
requirement for creativity and critical thinking
with
math avoiders who habitually avoid such requirements?

Three suggested formats?

- word problem for upcoming quiz
- discussion of effort in doing a problem
- essay/report from mathematics history

Occasionally assigned topic?

- from text’s “discussion questions”
- from daily life in Ketchikan

Many of my students have chosen, rather than using the UAS-K computer lab, to obtain email/internet access through commercial avenues. As a result, my own professional portfolio now contains a letter of commendation from the owner of an international service provider. That owner wrote enthusiastic support for the spirit and construction of my email assignment--as both a motivator for students to understand mathematics and as a "real" assessment of critical thinking skills. (I'm sure it didn't hurt that it brought business to the firm.)

Note that I do not accept: articles pasted off the web; articles without a vocabulary word used in context; late submissions. I do accept hard copies only in the case of server difficulties (which of course occur regularly). For their own protection against "lost in cyberspace, which happens, like it or not. Here's the coping strategy", I also heartily suggest that students set-up their accounts to "save on send" and/or to copy themselves with each submission.

"I am not their English teacher." However I do value meaningful and efficient communications. While I do not grade their grammar and spelling, I do frequently correct it and offer suggestions for improvement. My main concern, as indicated by the all or nothing grading criteria, is that students attempt and practice technical communication. This in itself is quite intimidating and difficult for many students. That "all I want is for you to try" allows many new students to overcome insecurities related to both their communication and computer skills. The goal is to practice this skill (technical writing), and by the practice to improve.

Many students use this assignment to have fun and write about mathematically related life, jobs and jokes. Others write reports. Some try to format data--and learn that some email is still not good at non-standard presentations/spacings of text. Some sigh heavily, not quite triumphantly, at actually having completed and sent three sentences on a machine. Several "foreign" students have used the journal to practice their writing skills while learning about Western units of measure, from the kitchen to the shipyard. It is a beneficial exercise for students at all levels of intellectual experience and ability.

Of all the the things I do as a teacher, none gives me greater insight into the mathematical needs, abilities, and interests of my students as individual learners, than does this weekly writing assignment. This is a win-win assignment for teacher and students.

Note: The overall course grading rubric here demonstrates some of that mathematical density. The ease with which this writing assignment is scored reflects the importance of practicing technical communications skills, while it also partially offsets--in the big picture--low scores typical on my tests and quizzes. So I depend, in part, on this numeric counterbalance to “enable my tests and quizzes to discover the limits of student ability.” (Or in students terms, my quizzes and tests are “thorough.”)

The fourth and final realm of assessment, the portfolio, is also an “easy A” in the grade book. All it requires is work. This particular work is all very mathematical in nature, and very practical for career applications.

I have seen very many descriptions and definitions of what a portfolio is to be, and can be, in the mathematics classroom. Here is the outline which I have come to prefer, especially for entry level adult students; all students receive the list and requirements on day on.

Log of Logs: the portfolio

Job Skills and Good Study Habits

Documentation Practice:

applied to their own activity

Ability to follow instructions:

applied to what has been declared
important

Ability to organize information:

into not-necessarily discrete categories

Individualized resource centers:

Open for use during Exams
Information for “self-grading” algorithm
Reference in future courses

Log of Logs: the portfolio

DENSITY

While focused on traditional
mathematical content, this assignment
addresses:

Self Assessment
Information Literacy
Manipulation of information formats
Communications skills
Critical thinking
Professional behavior

Class-time: minimal, only for
clarifications

Teacher time:

15 minutes per student at midterm:

interviews

<10 minutes per student at end of term

List of required logs

- 1) Table with chronologic listing of i) all ASSIGNMENTS given, and ii) the date assigned
- 2) Table with chronologic listing of the i) the main topic, and ii) the date submitted for each E-MAIL submissions.
- 3) Students will compile a comprehensive log of i) all QUIZ and EXAM questions, with ii) correct solutions supplied for each.
- 4) Students will keep a log which i) lists and ii) gives the context for the “TOOLS of the TRADE” introduced and/or used in the course.
- 5) A listing of all technical words / VOCABULARY introduced in class and in the text.
- 6) A listing i) all mathematical SYMBOLS, including ii) a description of their meaning and iii) appropriate context for use.
- 7) A comprehensive log which i) lists CALCULATOR USES, and ii) gives details for commands and chains-of-commands associated with these uses.

8) A compilation of i) WRITTEN SELF-ASSESSMENTS
and ii) selected Examples of homework.

[Having briefly described the list of logs and their contributions to the overall course and course grade, the session Presider cordially gave me the five minute warning.] Since it was Sunday morning and no sessions were scheduled to begin after mine, and since my proposal included discussion of manipulatives and cooperation in the entry level college classroom, I decided to forge ahead, and give these last two topics a minute each before wrapping up.

There seem to be two somewhat distinct interpretations of the word “cooperation” when applied to the classroom setting. Neither need be used independent of the other, especially when working with adult students. First, a general and underlying cooperative attitude is a beneficial tool whether in a traditional classroom (recall the Dr. Ballard story!) or an interactive one. Second, there are many structured cooperative activities designed to illustrate concepts and properties from mathematics.

There are many approaches available to guide the formation of structured groups--all of which are highly mathematical operations. From these, I prefer to let density be the deciding factor--given the particular collection of students, learning goals, etc. I use random grouping almost exclusively at least until every student has had the opportunity to meet every other student. Random grouping is especially beneficial with adults, especially those with dreadful recollections of past excursions into math class. (Traditionally, looking at the clock to help visualize a circle problem is considered cheating. Now they're being asked to share with and use each other! Older students are especially disconcerted by this shift, even in 2000.) I prefer to use the standard deck of playing cards to obtain random grouping assignments--density again...cards can be used in many and diverse mathematical settings...last year I did a workshop with them, and I use them alot in the classroom...but that's a different story

In the handout (of shrunken copies of the overheads) there's a grading algorithm for group projects that is densely mathematical and very easy to use. I call it “The final page.” It's guaranteed fair to each student, and easy to figure.

[And this is about all I said as I flashed two of the following overheads on cooperation. Then I flashed into a quick look at appropriate use of manipulatives.]

Cooperation in Learning

(see Dees, Crabill, Burns, Terwel, Weissglass)

Individuals with a common goal

- ↪ An underlying attitude
- ↪ Especially beneficial with
advanced, slow, and non-traditional students
- Adaptable to many learning and teaching modes
- Effective for higher order content investigations
- The Normal Curve doesn't apply, don't use it !!!!!
community college students are not a
random sample of the "normal" population.

Cooperative Classroom Management

(see Slavin, Johnson and Johnson)

Individuals arranged in groups

- ↪ Highly structured; include competitions
- ↪ Especially helpful for: feedback in large classes;
under-skilled, widely diverse students

- Efficient for many tasks (esp. data entry!) and projects
- Effective with skills based, mastery activities

Cooperative Structures

Density through randomness

content
manipulation and psychology

My preference is randomly assigning
students to small groups to work.

Less indignation and/or resistance by adults
for being manipulated
("like a child or plaything")

Enables each student to meet all other students,
learn names, without intrusiveness.

Leads to a more comfortable workplace.

Enables large comparison group for self-assessment.

**

Has led to students finding others they work well with,
leading to out-of-class cooperation and support systems.

**

Cooperative Structures

Content centered benefits

Direct, non-threatening experience with
randomness in action,
and randomizing devices.

Concrete, experiential basis on which to build
mathematical understanding of the concept of
randomness.

Focus for raw data: collection and analysis techniques.

How many different groups are possible in this class?

How many randomly assigned groupings:

Are needed until everyone has worked with everyone
else?

Did it actually take until everyone worked with
everyone else?

The U.S. Coast Guard model

for navigation personnel on the bridge

(Minimizes errors in tight waters.)

One reads out,
one reacts to it,
one watches over both, for typos in communication.

What is important?

How can it be measured?

How can we
reduce it to the realm of number?

The “FINAL PAGE” of any group assignment must consist of the following:

1) Explicit written description of each person’s contribution to the group effort, including but not limited to:

- scheduled, organized meetings
- focused questions, contributed key points
- prepared and/or researched
- secretarial and artistic contributions

2) A numeric measure of each person’s contribution. What percent of the entire project did each person contribute? How was this number determined?

3) Signature of each person, to show each accepts the breakdown given in part two.

Attach dissenting opinion if applicable.

Grading via the Final Page

Group assignments are graded as a unit, for x points.

These points are distributed to individuals proportionally,
as determined in step two above.

Density in The Final Page's

assessment of projects.

Self-Assessment

Content-area writing

Construction of random variable--to "budget" their
points

Meaningful "Subjective evaluation" skills

Meaningful percentages

Personal responsibility

As promised in the proposal and description of my presentation, the last main pedagogic tool on which I have results relates to manipulatives. Here again I note that I am the only faculty at UAS-K, and teach everything. From the vast array of manipulative educational materials available, I use primarily those that can be applied to multiple settings, those that enable students to find depth in mathematical analysis, and those that cost little money and take little time. My choices are further constrained by the non-traditional student base.

In the history of the human race, most of the mathematics ever developed derived in our desire to manipulate the world and materials around us. Manipulatives are an essential tool in the math classroom. They enable students to gain a beginning appreciation for the roots of mathematical theory, and the realms of application for useful mathematics. We are in a time when skill with mathematics is being required of all college students, no longer just the talented 10%. A widely used human growth progression works from the concrete example to the abstract representation then to the abstract model; manipulatives provide all students with an accessible foundation for this mathematical construction of knowledge.

A helpful tool should not, however, replace the ends. The goal is to teach mathematics skills and concepts, manipulatives need to effectively and efficiently contribute toward that end. And my time is just about up.

Again, the handouts include some examination of the concept and phenomena of mathematical manipulation. In particular I've found buttons extremely useful in teaching arithmetic, especially relating ratios to percentages. For higher level courses I use fish crackers, geoboards, and a lot of playing cards (see the Proceedings from last year for some of my work with cards in algebra).

Manipulation ...

of things to get at arithmetic

of numbers to get at algebra

of algebra to get at calculus

of things to get at discrete methods

of models to get at design

of measures to get at science

for conceptual understanding.

For real job skills.

For safe classroom:

investigations and concept development

mathematical modeling

remediation and the meaning of

division ...

Manipulatives.

Manipulative Properties

“ ... for every thing there is a time and a purpose ... ”

Specialized designs: single, well-defined use

one topic of study, one specific technique

“pick your topic, we’ll make you one”

Flexible designs: serve multiple purposes, multiple settings

integrate topics, adaptable to diverse techniques

“find them in the annals of human invention”

Everything in between

Time, Space, Energy, Cost Concerns

Let students do all the daily logistics ...

is good quantitative, logical exercise and experience,

motivated by classroom goals

quicker than one person (teacher?)

personal responsibility with numbers

Density and Manipulatives

For Preparatory Mathematics: M054

Buttons

jars and bags of *assorted* buttons

Inexpensive, easily stored, logistically sane

Count and classify a collection
according to:

Color (visual), one day;

Size (measurement), another day;

Hole design (conceptual), for the 3rd time through.

(Start with a different jar or bag each time)

Measure and Sort;

describe in words and/or drawings

Numeric comparisons

Fractions and Decimal parts of the whole,

Percentages

Results from different collections of buttons

Density and Manipulatives

For Probability and Statistics: M117 & S273

For Concepts & Applications of Contemporary Math:
M131

Cards

decks and games of standard playing
cards

Inexpensive, small, logistically simple

Conducive to small group investigations and data
collection

Useful in randomly assigning groups of ≥ 4

Full of content skills, concepts and
applications:

Set theory: notation, counting techniques

Multiple classifications and representations

Game Theory and the mathematics of competition

Probability, Statistics and Strategy

Definition and evaluation of random variable

Arithmetic and organizational skills

In summary, I've noted many diverse pedagogic techniques and mathematical topics have been added to our lists of tools and expectations as teachers of mathematics at the two year college. For the first time in history, all degree students must satisfy mathematics requirements. Our student population is not a random sample of a normal population. Many of our students have already failed and/or run away from mathematics under the traditional approach.

I have found that by paying attention to the interplay of technique and content--by making the class-time mathematically dense, in a user-friendly manner--we can approach a standards quality learning experience for our students, even when constrained by traditional administrations. I hope to have passed along a few of my successes in such an effort.

In a very practical sense as a full-time teacher, I have focused this task by interweaving and integrating five techniques: Content Area Writing, Portfolio maintenance, Self-assessment, Cooperative efforts, and Manipulation of things and measures of the things. The syllabus as a skeletal contract with students provides a powerful instrument to give weight to reform concerns, and to enable integrated assessment. The end.

There are some sets of self-assessment assignment forms, and copies of syllabi that follow the above outline, up here for those of you who'd like. As mentioned, this is a work in progress, so I'd welcome comments and suggestions--my email and address are on the handout.

[The original outline for this talk included 39 overheads. These were all reduced to fit eight on a sheet of paper, and given as a handout. Since $39 \equiv 7 \pmod{8}$, there was one-eighth page of open space to fill on the last page of handout. Thus the following page of philosophic commentary was included.]

“As little as a battle between
single ants of an anthill
is essential for survival
just so little is this the case
between members of the human race”

“In order to form an immaculate member of a flock
of sheep, one must, above all, be a sheep.”

Albert Einstein

“I have little patience with schemes, no matter how elegant,
that cannot be implemented by
a regular classroom teacher in a regular classroom.”

Roberta Dees
Purdue University

“To adhere rigidly to one method of small group learning
is as stifling and nonproductive
as to insist on the lecture method
as the sole method of instruction.”

Calvin D. Crabill
U.C. Davis

“When I look back on all the crap I learned in high school,
it’s a wonder I can think at all ... “

Paul Simon
Popular song

“Kodachrome”

