Teaching Calculus with the TI-92

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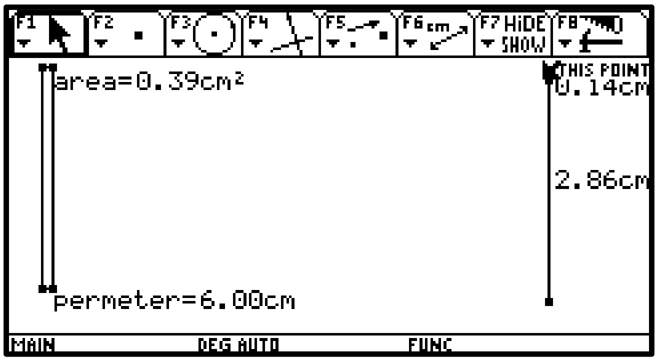
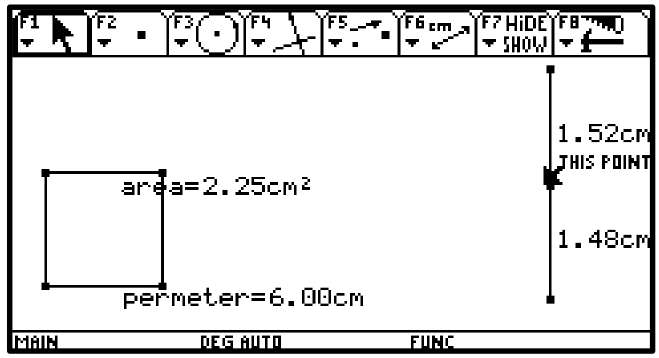
Maximizing the area of a rectangle with a fixed perimeter

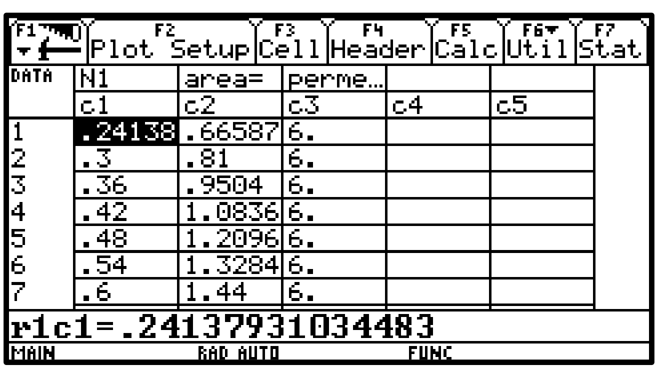
Using the Geometry Application on the TI-92, we can show students rectangles that have the same perimeter and different areas (I never feel students really understand what is happening when they do this problem). Then find the dimensions that maximize its area.





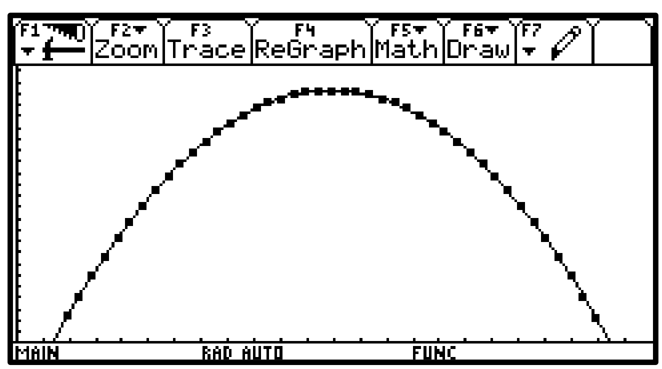
Create a rectangle of fixed perimeter 6.00 cm by using subsegments of the 3.00 cm segment at the right as its height and width. By moving the interior point on the 3.00 cm segment at the right of the screen, the area of the rectangle changes and approaches zero as its height approaches zero. By moving the point slowly when it is near the midpoint, we see that the maximum area of the rectangle is 2.25 cm ^{2}.

Although it is very easy to write the equation for the area [x (3 - x) = -x ^{2} + 3x], it is informative to animate the interior point moving on the segment and collect the height, area, and perimeter of the rectangle. Area can be plotted versus height to see that the data lies on a parabola. Power regression can be used to find its equation [Area = -x ^{2} + 3x]. Scrolling through the data shows that as the height increases, the area increases to about 2.25 and then decrease toward zero, and the perimeter remains constant.



DATA	N1	area=	perme…		
	c1	c2	c3	c4	c5
1	.24138	.66587	6.		
2	.3	.81	6.		
3	.36	.9504	6.		
4	.42	1.0836	6.		
5	.48	1.2096	6.		
6	.54	1.3284	6.		
7	.6	1.44	6.		

r1c1=.24137931034483



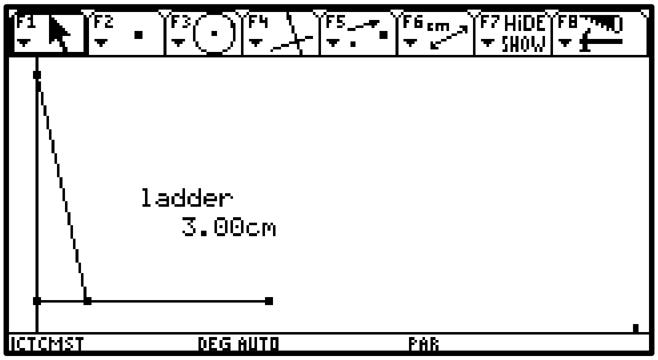
To use the derivative and CAS to find the maximum area,

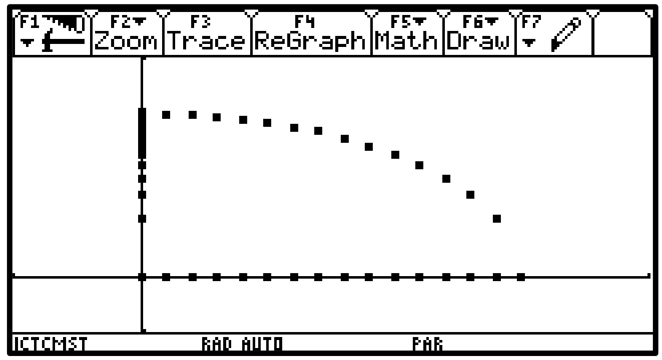
solve (d(x ⋆ (3-x), x)=0, x) x = 3/2

x ⋆ (3-x)|x=3/2 ≈ 2.25

Observing a Ladder Sliding along a Wall using Scripts

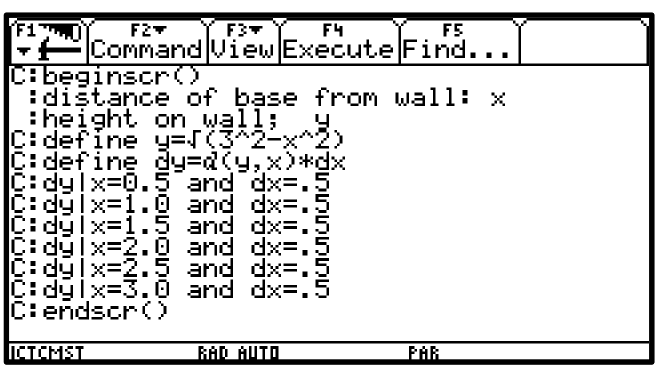
Use a geometry application to draw a ladder whose base can be moved along the ground in equal increments and watch the rate at which the other end slides down the wall. The ladder's length, 3.00 cm, was chosen to fit comfortably on the screen. To be realistic, 3.00 cm could represent a 12-foot ladder. Observe the rate at which the ladder slides down the wall; is it sliding at a constant rate or does it appear to be sliding faster near the bottom of the wall?

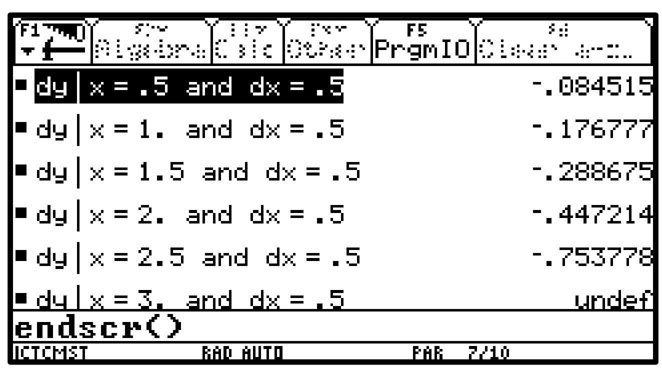




To see a different representation of this motion (based on a script written by Dr. Paul Beem, Indiana University at South Bend), let t represent the distance of the base of the ladder from the wall and $s(t) = \sqrt{(3^2 - t^2)}$ the height of the ladder on the wall. Graph the functions xt1(t) =t, y1(t) = 0, x2(t) = 0, y2(t) = s(t), x3(t) = t, y3(t) = s(t) in the parametric window [0,3] x [-1, 4] x [-1, 4] with t-step = 0.2. The first (horizontal) function gives the positions of the base of the ladder along the ground (equal increments), the second (vertical) function the positions of the top of the ladder sliding along the wall, the third (2-d) function the height on the wall versus the distance along the ground. The second and third functions show that the changes in height are greater as the ladder moves away from the wall.

To see the changes in height, use the script below to see the differences in the heights of the ladder as it is pulled away from the wall. The negative signs show that the ladder is falling; the increasing absolute values of dy show that it is falling faster as the ladder is pulled away from the wall.





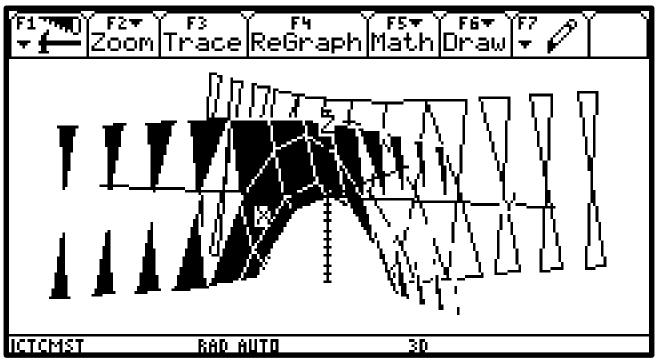
Graphing in 3-D

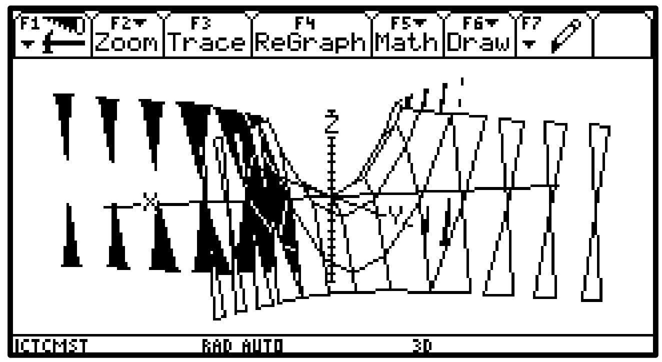
To graph the hyperbolic paraboloid $z = x^2 - y^2$, set the graph MODE to 3-D. Press ♦ F to change the format to show the axes and label them. Enter x ^ 2 - y ^ 2 for z1 in the Y= menuand press Zoom Standard [F2, 6] to graph. The shading shows the under side of the surface. Using Window, we see the first two entries in zoom standard are eyeθ° = 20 and eyeϕ° = 70. These are the cylindrical coordinate variables θ and ϕ:

θ is the angle in the xy-plane measured counter-clock-wise from the x-axis;

ϕ is the angle measured down from the positive z-axis.

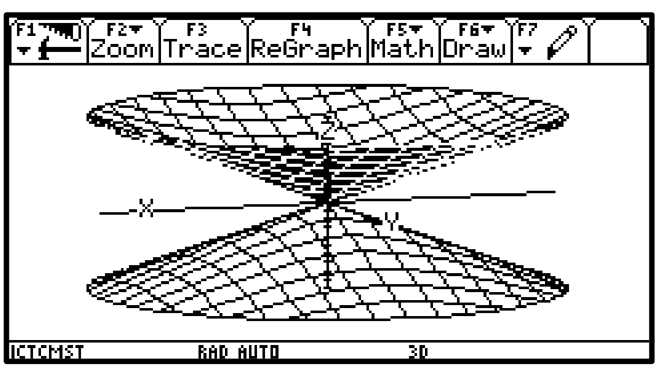
To rotate a 3-d function, change eye θ° and eyeϕ° in Window.



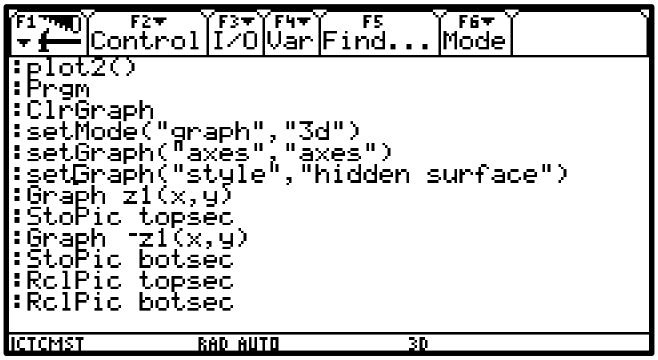


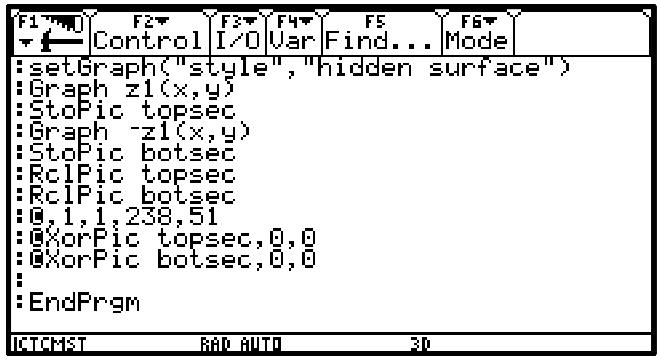
Try eyeθ° = 70 and eyeϕ° = 70 for a better view of the hyperbolic paraboloid (saddle surface). In this view you can see two intersecting parabolas in white: one opening upward, $z = x^2$, and the front part of one opening downward, $z = -y^2$.

A program, written by three of my students at Orange Coast College (David Brady, Chris Rounds, and Scott Lusby) allows us to see both nappes of the cone, $z^2 = x^2 + y^2$. Enter z1 = $\sqrt{(x^2 + y^2)}$. The program plots the function z1, saves the graph as a picture, then plots the function -z1 and saves this graph as a picture also. Finally it recalls both pictures to display the complete cone and the axes.



The program below, plot2, graphs both nappes of the cone, $z^2 = x^2 + y^2$ if $z1 = \sqrt{(x^2 + y^2)}$.





The program can also be used to plot the sphere $x^2 + y^2 + z^2 = 64$. Note that the function $\sqrt{(64 - x^2 - y^2)}$ must be entered as z1 to use this program.