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What Can I Do With the TI-92 CAS-Based Calculator in Algebra?
Session W19

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Consider the following situations and find a mathematical model for each.

a) The level of the drug Imipramine in the blood of a patient rises at a constant rate (for example 60 nanograms per week) until the patient is at the prescribed level and then the rate of change remains at 0% until the patient is taken off the drug at a constant rate (for example 90 nanograms per week).

b) The relationship between the medical charges filed by a subscriber and the medical charges paid by an insurance company that pays at a rate of 0% for the first $200 in medical charges filed and pays at the rate 80% for charges over $200.

c) The relationship between time and height of an airplane that ascends at a constant rate for 30 minutes, levels off at a 0% rate of change for 60 minutes and then descends at a constant rate for the last 30 minutes of the flight.

d) For users of electricity who also use a heat pump, many power companies charge, for example, $0.08 per kWh used for the first 1000 kWh and $0.05 for the next 1000 and finally, $0.03 for any consumption over 2000 kWh.

e) Piece-work salary and sliding scale commissions are business examples of this same structure, and are used to provide incentive for higher employee productivity. For example, a business may pay the sales staff 3% commission on sales from $0 to $10,000, 5% on sales from $10,001 to $15,000, and 8% commission on sales $15,001 and over.

f) The pay scale for piece-work by a telemarketing company is $0.35 for the first 300 calls in the week, $0.42 for the next 200 calls, and $0.65 for any call over 500 calls.

g) A new long-distance phone company has the following rate schedule:
12¢ per minute for the first 15 minutes
9¢ per minute for the next 10 minutes, and
6¢ per minute for any time over 25 minutes.

h) The 1994 (or any year) federal income tax form 1040 schedule (for single filers) had rates of taxation of 15% on the first $22,750 of taxable income, 28% on the next $32,350, 31% on the next $59,900 of taxable income, etc. That is, the taxable income brackets are at $22,750, $55,100, and $115,000. (There were two more brackets that will be ignored for the sake of brevity.)

i) A commercial airline flight from Reno, Nevada to St. Louis, Missouri ascends at a constant rate (after initial take-off) of 1100 feet per minute for 30 minutes (until it reaches a cruising altitude of 33,000 feet). It then levels off until it is 60 minutes into the flight. At 60 minutes into the flight, it has burned off enough fuel to ascend to 37,000 feet at a rate of 400 feet per minute. This takes 10 minutes. The plane remains at 37,000 feet until 175 minutes of flight time when it descends at a rate of 1233 feet per minute and then lands in St. Louis.

The Algorithm for Finding the Model.

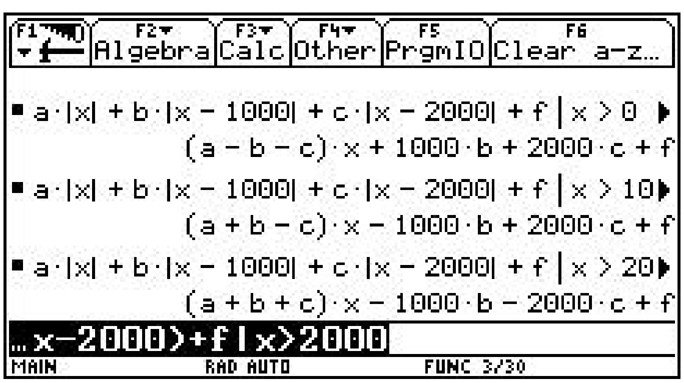
The algorithm assumes a knowledge of behavior of sums of absolute value functions (Laughbaum, 1996).

1. Recognize that the structure of the model for all of the above situations is $M = a|x+e_1| + b|x+e_2| + \ldots d|x+e_n| + f$, where there are n corners.
2. Find the corners (e_1, e_2, …) [corners are normally known]
3. Simplify the model for each rate interval using the TI-92
4. Set the coefficients of x equal to each rate of change. [rates of change are normally known]
5. Solve the system.
6. Find f using a geometric transformation of a vertical shift.

Two Examples:

1. For users of electricity who also have a heat pump, many power companies charge, for example, \$0.08 per kWh used for the first 1000 kWh and \$0.05 for the next 1000 and finally, \$0.03 for any consumption over 2000 kWh.

1. The model is $C(x) = a|x+e_1| + b|x+e_2| + c|x+e_3| + f$ [There are three corners.]
2. The corners are at 0, 1000, and 2000 kWh used; thus $e_1 = 0$, $e_2 = -1000$ and $e_3 = -2000$.
3. This model simplifies as follows:



$x > 0$ and $x < 1000$

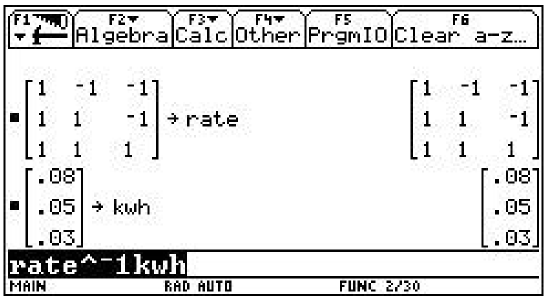
$x > 1000$ and $x < 2000$

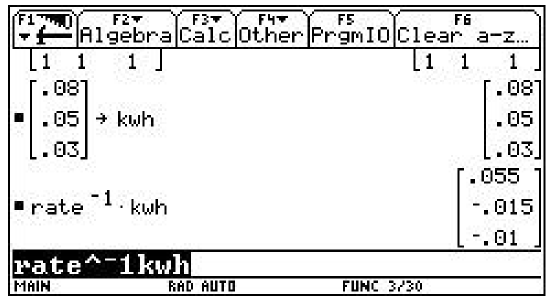
$x > 2000$

4. Since the function has a graph that is linear on each rate interval, the coefficient of *x* in the above simplifications is the given rate of change in the problem.

$$a - b - c = 0.08$$
$$a + b - c = 0.05$$
$$a + b + c = 0.03$$

5. The solution is:

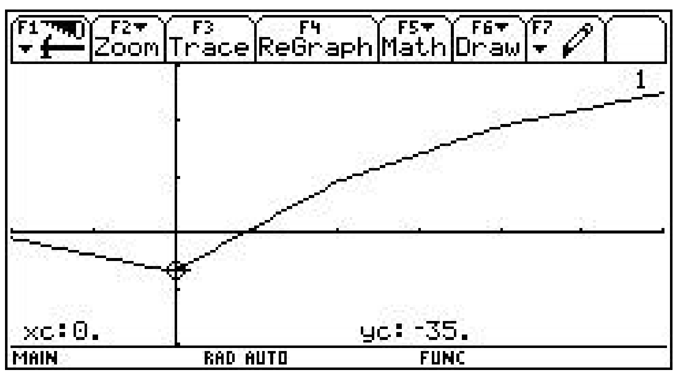




That is, a = 0.055, b = -0.015, and c = -0.01, or

$$C(x) = 0.055|x - 0| - 0.015|x - 1000| - 0.01|x - 2000| + f$$

6. Find *f*. The model is at -35 when *x* is 0, So, add 35 as the value of *f*.



The final model is $C(x) = 0.055|x - 0| - 0.015|x - 1000| - 0.01|x - 2000| + 35$.

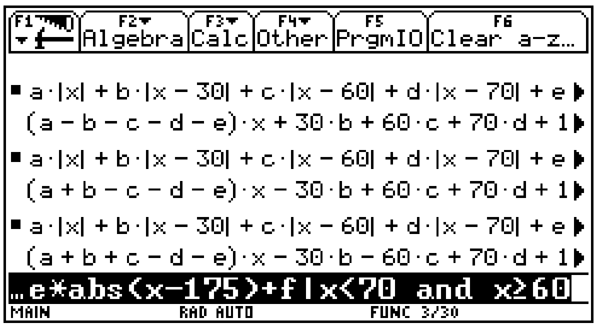
2 The on-board computer on a commercial airline flight from Reno, Nevada to St. Louis, Missouri commands the plane to ascend at a constant rate (after initial take-off) of 1100 feet per minute for 30 minutes (until it reaches a cruising altitude of 33,000 feet). It then levels off until it is 60 minutes into the flight. At 60 minutes into the flight, it has burned off enough fuel to ascend to 37,000 feet at a rate of 400 feet per minute. This takes 10 minutes. The plane remains at 37,000 feet until 175 minutes of flight time when it descends at a rate of 1233 feet per minute and then lands in St. Louis.

Recognize that the structure of the model is

$$h(x) = a|x+e_1| + b|x+e_2| + c|x+e_3| + d|x+e_4| + e|x+e_5| + f$$, where there are 5 corners.

1. Find the corner parameters (e_1, e_2, …). They are given as 0, -30, -60. -70, and -175. Thus, the initial model is $h(x) = a|x| + b|x-30| + c|x-60| + d|x-70| + e|x-175| + f$, where x is time in minutes.

2. Simplify the model for each rate interval using the TI-92

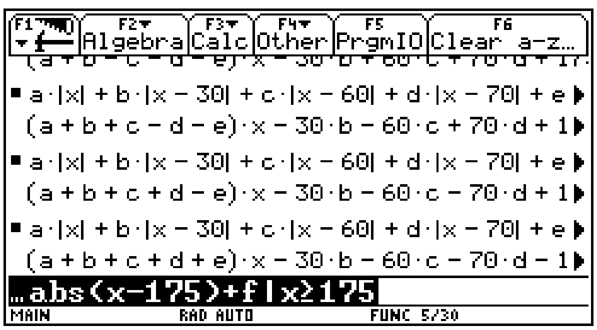


Conditions

$x \ge 0$ and $x < 30$

$x \ge 30$ and $x < 60$

$x \ge 60$ and $x < 70$



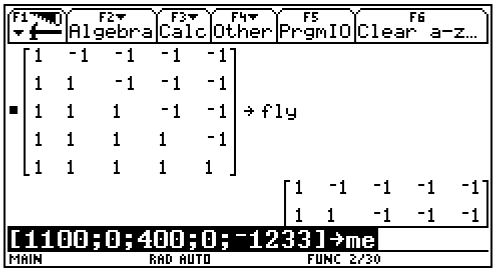
$x \ge 70$ and $x < 175$

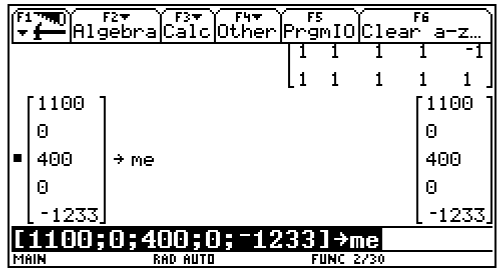
$x \ge 175$

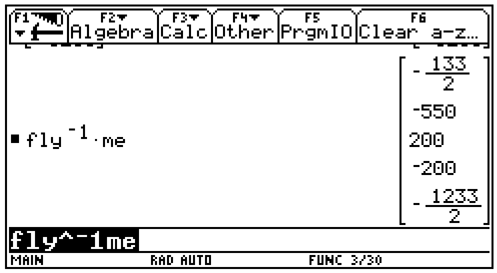
3. Set the coefficients of x equal to each rate of change; that is, each piece of the model simplifies to a linear function as shown on the TI-92 screens. Thus, the coefficient of x is the rate of change for each piece.

$$a - b - c - d - e = 1100$$
$$a + b - c - d - e = 0$$
$$a + b + c - d - e = 400$$
$$a + b + c + d - e = 0$$
$$a + b + c + d + e = -1233$$

4. Solve the system.





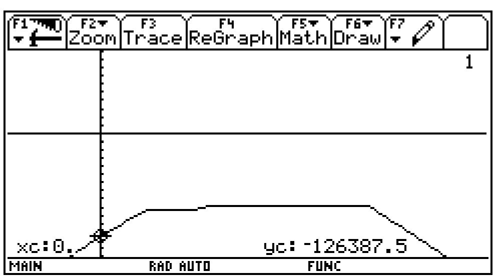


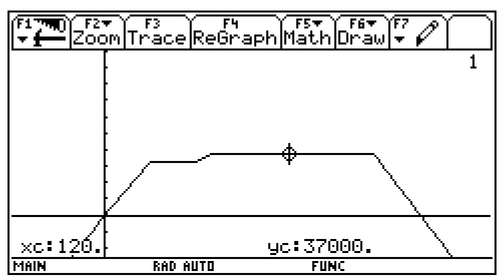
The model for the height control function is

$$h(x) = -\frac{133}{2}|x| - 550|x-30| + 200|x-60| - 200|x-70| - \frac{1233}{2}|x-175| + f\,.$$

5. Find *f* using a geometric transformation of a vertical shift.

When *x* is 0, for example, the function is -126387.5. Add 126387.5 as the parameter *f*.





The final model needed by the computer is

$$h(x) = -\frac{133}{2}|x| - 550|x-30| + 200|x-60| - 200|x-70| - \frac{1233}{2}|x-175| + 126387.5\,.$$

References

Laughbaum, E. D. (1996). Modeling data exhibiting multi-constant rates of change. The AMATYC Review, 17(2), 27-34.