Multimedia in the Mathematics Classroom

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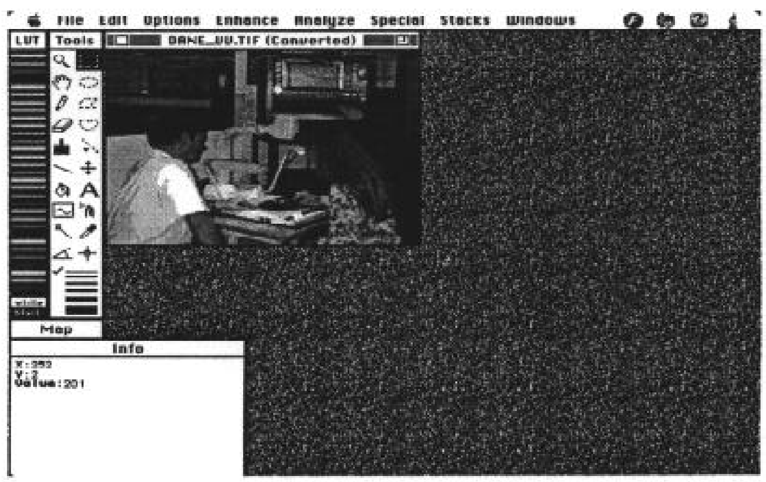


Multimedia in the Mathematics Classroom

The teaching of undergraduate mathematics is currently undergoing a major transformation in response to the efforts of AMATYC, the National Research Council, MAA, NCTM, and other professional organizations. The National Research Council, in *Moving Beyond Myths: Revitalizing Undergraduate Mathematics, [1991]* stated "The way mathematics is taught at most colleges (lecture) has changed little over the past 300 years, ... , detached from the life experiences of students and seen by many students as irrelevant."[1] The emphasis in undergraduate mathematics will change to improve the transfer of skills between disciplines, the motivation of students, retention of knowledge, the synthesis of concepts, and tools for the solution and modeling of real problems even at the developmental mathematics level. Projects currently funded are attempting to revitalize and redefine instructional and teaching methods through the use of multimedia technology.

The use of multimedia in the mathematics classroom of the future will allow for the integration of mathematical topics and concepts with practical application, modeling, and data analysis. The application of these concepts with diverse student populations and levels, through the use of technology and multimedia presentation, will emphasize the active learning environment and a collaborative atmosphere created through the variety of techniques currently available. The use of technology assists students of diverse backgrounds and learning styles, particularly in developmental mathematics, to develop a cognitive visualization of the concepts and therefore improve both their skill with manipulation and the transfer of these skills.

Multimedia techniques and the use of available technology in the classroom allow for the ideal visualization of difficult concepts, clear understanding, and the exploration of math-ematics. The effective integration of media can enhance the learning environment and assist both the student and the teacher. Cognitive research indicates the majority of us are visual learners - that we must form a picture in our minds to assist the comprehension of concepts. Traditional methods do not take advantage of this fact, and our new standards for mathematics education as written in the AMATYC *Crossroads* and NCTM *Standards* documents emphasize the need to change the emphasis on these traditional methods. All students have a better chance of becoming an effective problem solver and develop mathematical power if abstract concepts and principles are presented visually. Never before has there been the media for supporting this premise. The equipment and techniques described in this paper are commonly available and include presentation, CD-ROM, laser disc, image processing, spreadsheets, and graphing calculators with the CBL. Visualization techniques and approaching mathematical concepts numerically, graphically, and symbolically will be the emphasis of this paper.

Multimedia:

The term multimedia refers to a complete delivery and environment system which integrates the text, graphics, animation, data, video, and audio from various sources which can include video-disc, CD-ROM, video visualizers, graphing calculators, and the computer itself. The computer managed system can allow the instructor to design an instructional plan for courses which assist the student in interacting with other students and the instructor, explore the desired concepts, learn skills and principles, and produce results of problem solving utilizing the media available. The computer itself is now a self-contained, interactive multimedia machine which includes hypertext, buttons, video, and audio in the working environment. It easily interfaces with other media equipment, making it now possible to design a mathematical learning environment with a "high level of interactivity, simple structure, effective navigation tools, and an upgradable format."[2] This system can include a library of

1[Moving Beyond Myths; National Research Council; 1991]
2[Multimedia Mathematics; Margarita/Salinelli; Available [online] at: http://www.lib.siu.edu/itvc96/papers/5/;10/29/96]

terms, manipulations, or functions and an instructor can design labs, exercises, and tests utilizing the available media.

The modern mathematics department will have hypermedia media production equipment available for faculty use. The equipment will include production media for presentation video of still and moving images, hyperlinks and computer communication, laser disk and camera support, and other media. Additional libraries of CD-ROM titles, laser discs, and software titles will support the production of an environment tailored to the student, faculty, and institution. The system will be computer managed and include the standardization of the programming environment, to enable the faculty to easily utilize available media.

Multimedia Equipment and Applications:

Many institutions currently have or are planning multimedia classrooms for mathematics and other disciplines. As an intermediary measure, many also make use of the COW (Computer On Wheels) which is easily transportable to any classroom. Difficulty with projection in the traditional classroom has led many institutions to invest in the planning of classrooms designed specifically for presentation with multimedia and computer laboratories which support this learning environment.

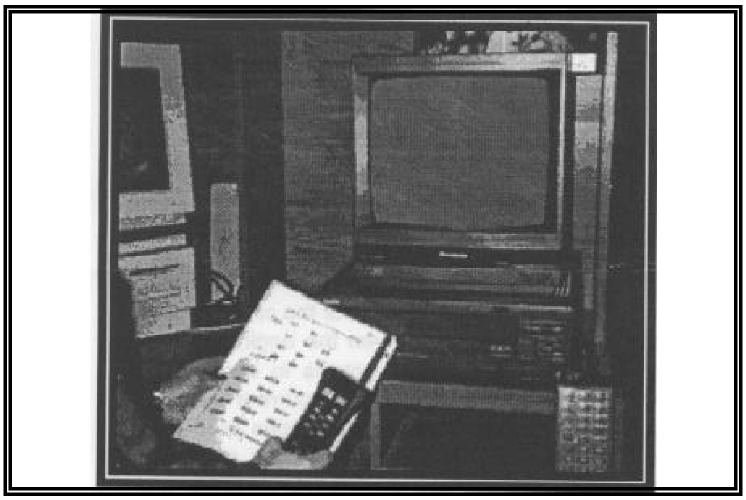
The following descriptions are by no means complete, as there are a myriad of applications which are available to support the production of multimedia mathematics presentations. They are, however commonly available on most campuses (although perhaps not within the mathematics departments).

- **Presentation Software:** Use of software such as PowerPoint (Microsoft), WordPerfect presentation, and others make the organization of classroom presentations and application of the use of still and moving images, note-taking and other class information a breeze. Templates for "slides" make the placement of text, images (still and movie clips), and coordinated audio faster and easier to produce. Color overheads, slides, speaker's notes, outlines, and handouts are all included in the software's design. Through the use of tools and different views, shortcut buttons, and previewing your presentation enables you to design and revise classroom materials easily. This works effectively for materials and topics you tend to use frequently, perhaps in several classes, with minor adjustments. Collaboration among faculty can produce dynamic results.

- **Interactive CD-ROM:** The production of good interactive CD-ROM for undergraduate mathematics has been slow and some of the CDs available are no more than lecture on CD, but recently this has improved, with more in current production. An interactive CD-ROM allows the student to interface with the computer through the use of menus, toolbars, and buttons. The student can choose to explore a specific topic or chapter of material, progressing at their own pace. Additionally, the student can usually use within text hyperlinks, allowing them to jump to pages where the specific term or concept is further modeled or defined. On the CD-ROM, text is integrated with graphics (plots or geometric representations of mathematical concepts), and the student is able to change the position of the graph, change the values of a function or table, and input other information to explore how the numerical and symbolic changes affect the visual picture in the simulation.

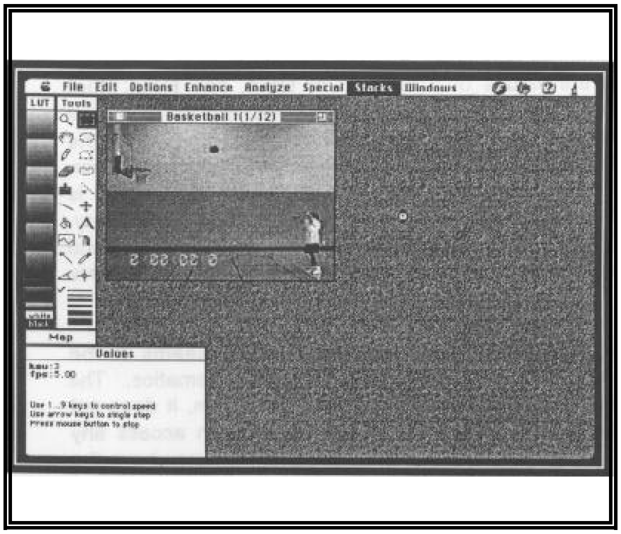
Also, video clips of events which are difficult to reproduce in the classroom setting can often be included on the CD-ROM. Students can go back, go over, and delve deeper into the topics included, which is difficult to do unless the student is working individually. The CD-ROM is easily transported in the classroom, laboratory, or made available to any student. The Algebra and Geometry software by Broderbund and Interactive Calculus by Larson, Hostetler, and Edwards are good examples of interactive materials.

- **Laser Disc:** The laser disk player is a piece of equipment which seems to be underutilized, perhaps due to the lack of many good laser disks for mathematics. The laserdisc contains video clips, slides, or movies and unlike videotape or film, it does not deteriorate with time. A major advantage of the laserdisc is that you can access any frame in less than a second, if you have the CAV (Constant Angular Velocity) laserdisc. In this format, each side can have up to 54,000 frames for you to choose from. The laserdisc player operates in manual mode with the user in control of the player, using a hand control or barcode reader. The disc can be accessed by frame, chapter, or time, and easily allows you to step forward or back and repeat the sequence of frames shown.



These discs can be used without a computer, and the user can use computer barcode production software to produce barcodes in any desired sequence of presentation. Two excellent examples are the Puzzle of the Tacoma Bridge (mathematical modeling) and Technology in Mathematics Education (teacher preparation) by Pioneer, among many others. Many science laserdiscs will support or provide a visual model for the mathematics topics you present. Students enjoy running the laserdiscs while you provide the explanation, often providing a way to involve an inactive student.

- **Image Processing:** The use of image processing has recently been enhanced by the production of classroom appropriate materials by the CIPE (Center for Image Processing in Education) in Tucson, AZ. Image processing involves scanning, enhancing, and measuring both still and moving images, using tools to change pixel values and locations in order to more clearly analyze the data represented in the image. The NIH (National Institute of Health) image software has been enhanced to make the processing of images easy enough for even young students. Classroom teachers have produced laboratory explorations which enable the student to learn about image processing (one of the major computer functions for the future), practice prediction and visualization skill, produce graphs and charts of data from images, and analyze the results to produce models and solve practical problems. Images from everyday events, space and earth science, biology, physics, and commonly encountered algebraic and geometric applications of statistics and mathematics are used.



Faculty can use the materials available and produce their own images for use to support their lessons. The use of hands-on images takes the student beyond knowledge to application and synthesis of information. The student can measure an area on a map, produce a density slice or surface plot for comparison, select data points to collect, and compute the model of resultant waves or the path of a trajectory. The curriculum material objectives support teaching within the national standards. The basic Image Processing in Education and new HIP series: Biology, Physics, and Educators, as well as the Developmental Mathematics products now in production are supported by professional development programs. Available in both the MAC and IBM platform, image processing provides the use of TIFF files, videos, and mathematical modeling within real science application, as well as the opportunity to improve a student's computer literacy.

- **<u>Spreadsheets:</u>** The use of spreadsheets in undergraduate mathematics will increase with the emphasis placed on the use of real data and analysis. The use of the spreadsheets available to interface with wordprocessors, such as Excel, allow even the beginning level student practical experience with placing appropriate tables and graphs in documents, providing supporting evidence of predictions or statements. The most current versions of spreadsheets allow easy access to tools for finishing the charts and graphs produced, and allow the use of patterns and functions in cell operations. Any data the student is assigned or acquires can easily be placed in a worksheet and descriptive statistics and graphs for the visualization and comparison of data quickly produced. The testing of hypotheses or predictive models are also supported through the use of spreadsheets. The practical application of financial, statistical, and other mathematical formulas can be performed on data in worksheets. Students can have the opportunity to explore variations in the data and the results of those changes, by writing functions or applying built-in functions. The student's comparison and communication skills are enhanced by observing spreadsheet data and interpreting the results of cell operations and functions.

- **<u>Graphing Calculators and CBL:</u>** The use of the graphing calculator in the mathematics classroom has become fairly common, with many mathematics departments requiring the use of the graphing capabilities in developmental mathematics. Manipulation of symbolic expressions and their graphs to visualize and predict the resultant change is done inexpensively through the use of the graphing calculator. The major advantage of the calculator is its portability and with a computer interface, many applications are possible.

  The behavior of functions and data, producing models, and programming are all important, meaningful applications performed with the graphing calculator. Calculators can be used to explore, test conjectures, and support conceptual understanding. Applications modeling and problem solving can be enhanced with the use of the CBL (Calculator Based Laboratory) unit. Using this portable unit, the students can gather data on temperature, light, motion, pH, and other variables in experiments, explore the variations of real data, produce models, and finally explain and confirm mathematical concepts. These calculators and units are the most inexpensive and portable of available media and many materials are currently available in both laboratory activities and program format.

- **<u>Hypercard/Hyperlinks:</u>** The use of hypercard stacks can be accomplished through the use of available authoring software, such as Hyperstudio. Hypercard is used to create your own task design for integrating different tasks, such as gathering information and organizing it, browsing information, meeting specific needs, or customizing computer work space. You design and control the Hypercard environment, linking buttons to various stacks, creating scripts and a customized environment for exploration and information. The links allow immediate access to information or graphics, whether it's in that stack or not. Buttons, scroll bars, and objects are easily integrated into a Hypercard stack through the use of authoring menus. One precaution - the use of Hypercard is not recommended for material which may have to be frequently edited or is not used often, as the production of the stacks can be time consuming and often the tasks are more easily performed using other media.

  Hyperlinks are more commonly being used in the production of Web pages, to access internet information. Through the use of Hyperlinks (highlighted text), a student can easily access additional information from an internet search, Web site, or document reference. Current Web browsers have Hypertext production tools, readily available, which create the links as easily as typing and highlighting text. This can be used by students and faculty alike.

- **<u>Communication/Internet:</u>** Computer communication now takes many forms: e-mail, file exchange, access to databases, information retrieval, and others. Students can use the media for the exchange of ideas with students, faculty, and scientists to discuss topics and issues. Through the file exchange, they can access programs for the calculator and computer, and retrieve information via the many available databases. Students can add a global perspective to their projects by exchanging data and information with other students, to improve their writing ability and conduct experiments involving the sharing of data. Research using databases and other resources will improve the quality of the student's work at institutions where the on-campus resources are limited. The accessible resources include data, facts, images, sounds, freeware and shareware, among the other resources, including human.

  Through e-mail, a faculty member can communicate individually to answer a student's questions, set assignments and schedules, and leave other correspondence for students. Additionally, appropriate data and programs can be made available for students, including images and graphs for analysis. For the student who has difficulty meeting an instructor during office hours, this could be the media to provide contact. As distance learning grows, this type of communication will provide an essential link between the students and faculty.

  The citation of internet resources is currently under discussion, with rules for referring to resources evolving. Several good resources for citation are available in *Citing Internet Resources*; Classroom Connect's March 96 issue. The MLA Citation Guide [http://www.cas.usf.edu/english/walker/mla.html] and Citing Computer Documents [http://seal.ctstateu.edu/history/cite.html] are two online documents available. The recommended structure for citation is:

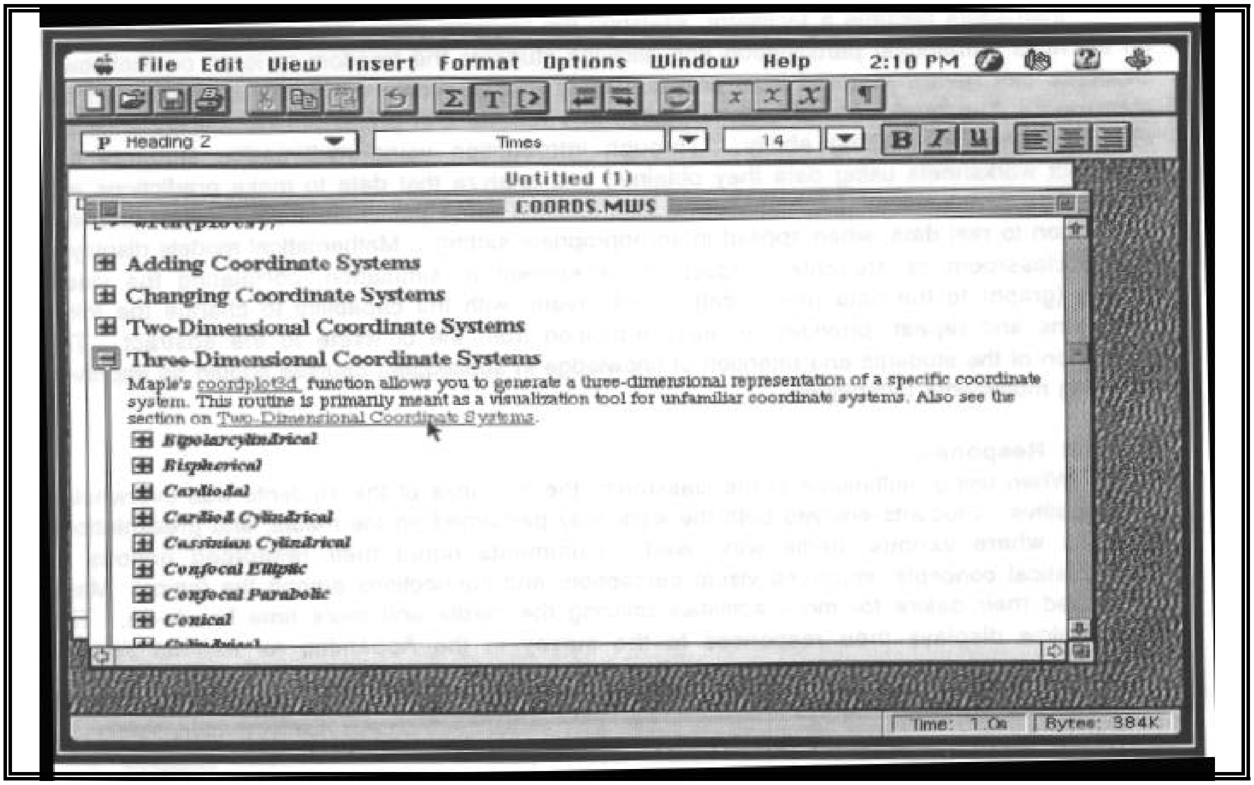
Author; Subject or Title; [online] Available ____ (e-mail, gopher, ftp:, etc.): address; path; date of document, session, download or posting[3]

Students (and faculty) should be made familiar with proper citation in their research and writing, as with any cited resource.

- **<u>Geometry Tools:</u>** The use of available geometry tools, such as Geometer's Sketchpad or Geometric Supposer allows the instructor and student to construct a variety of geometric models: plane and space figures, working models of geometric theorems, perspective drawings and tesselations, fractals, and trigonometric applications. Transformational geometry and topological notions are explored easily with this software. Students can manipulate the figures to discover patterns and relationships of geometric figures. The constructions lead naturally to generalizations as students see which aspects of the geometry change and which remain constant as they manipulate the figures, as all parts of the figures are updated continuously. The student can examine a set of similar cases, measuring quantities and expressions. Scripts to describe constructions can be recorded, to be used in your toolbox. Students (and you) can color, label, and annotate drawings. Additionally, animation, sounds, and film production are possible. Scripts can be annotated, saved, and printed to provide the student investigations and a forum for discussing geometric ideas.
- **<u>Digital Camera/Camcorder and Scanners:</u>** The digital equipment necessary for the production of your own images and video has become more cost efficient and accessible. The digital camera or camcorder plugs directly into a digital board on the computer (the digital board is necessary) to allow images to be imported into computer documents. The camera works essentially the same as an ordinary camera, using a video floppy instead of film, and immediate viewing of the images is possible. Images opened in processing software, such as Adobe Photoshop or NIH Image can be further enhanced and cropped. Video can be accessed by frame to enable the use of specific sequences.

  Since digital cameras are not as frequently available, images can be scanned using OCR graphics on a scanner, which can also scan text. Photoshop is among the easiest applications for use with a scanner, although processing and enhancing the images takes a great deal of practice. Menu items and tools allow you to acquire, display, and enhance the desired image. Image processing can be time consuming and a time effective technique is to use commercial or shareware images. Additionally, both platforms now support screenshots, which are extremely useful in the production of laboratory explorations and using mathematical computer applications. Screenshots are " snapshots of the desktop screen of the computer, showing all open windows of the computer application. These work effectively for placing in introductory documents for students. An example from Maple is given at the top of the following page. It allows the instructor to provide step-by-step directions and exhibit the resulting displays. This is especially helpful for the developmental or students with little computer experience.

[3][*Citing Internet Resources*; Classroom Connect; March 96]



The advantage of showing the display is the clarity of directions and immediate feedback for the students when they have appropriate results. Tailored assignments including the proper use of the media can now easily include screen images without the use of digital cameras.

- **Video Visualizer:** The video visualizer allows the visualization of models or equipment when it is not possible for all students to have the equipment or for viewing objects too small for the entire class to see. This equipment is expensive, but when the cost is shared by departments, the technology is useful for the projection of any object set upon the viewing surface, can enhance demonstrations of new calculators (students can view keystrokes), enable the viewing of small objects (zoom functions), and other purposes. While the benefits of this equipment is obvious in screening objects in science, it can also be useful in some mathematical applications.

Benefits of Using Multimedia:

The major benefit of the use of multimedia in mathematics is the high interactivity of the student and the mathematical concepts, as well as the practical application of the skills learned. Students use many senses and thinking skills, are readily able to extend their experience in an individual manner, and are provided immediate feedback in their explorations and results. Students can stop and explore, or repeat a process as often as they wish to, or go to another part of a program that offers a different kind of explanation, example, or function without limiting the progress of another student. Problem solving in collaborative groups and comparing results assists in the development of communication and team skills, as well as provide many more examples in order to generalize concepts.

Instructors become a facilitator, assisting the students when they need it most, focusing on activities demanding participation and allowing students the freedom to learn on their own. Students can review material from previous classes and events not easily duplicated in the classroom. The production of graphs pasted into reports and presentations will enhance the student's scientific writing ability. Through introduction using multimedia, students can construct worksheets using data they obtained and analyze that data to make predictions and appropriate conclusions. Mathematics is best learned when the student uses exploration and application to real data, when applied in an appropriate setting. Mathematical models displayed in the classroom as students conduct an experiment or simulation, correlating the visual picture (graph) to the data (numerical) of that event, with the capability to change the initial conditions and repeat, provides an easy transition from the concrete to the abstract. The motivation of the students and retention of knowledge in subsequent courses should be improved by using multimedia techniques.

Student Response:

When using multimedia in the classroom, the response of the students was overwhelmingly positive. Students enjoyed both the work they performed on the media, and presentations in class where various media was used. Comments noted their reinforced notions of mathematical concepts, improved visual perception, and connections among the topics. Many expressed their desire for more activities utilizing the media and more time hands-on. The chart below displays their responses to the survey in the Appendix, as well as specific comments.

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
I. TI-82					
a. easier to understand	33	62	5		
b. increased comprehension	38	57	5		
c. easy to use	14	57	29		
d. assisted visualization	57	38	5		
e. prompted purchase	33	19	29	9.5	9.5
f. increased interaction	24	52	24		
II. Multimedia Demonstration					
a. appropriate	24	76			
b. interesting	38	57	5		
c. good introduction	38	52	10		
III. Image Processing					
a. assisted familiarity	24	52	24		
b. interesting labs	19	57	19	5	
c. practical application	14	57	29		
d. increased time on task	19	48	33		
IV. Power Point Presentation					
a. more structured	10	52	33	5	
b. better attention	24	43	33		

c. easier note-taking	9.5	33	48	9.5	
d. prompted use of application	9.5	29	52	9.5	
V. Increase Willingness to Enroll	38	38	24		
More Supplemental Labs	38	48	14		

Examples and Ideas:

The examples which follow in the appendix are classroom tested ideas used to enhance the learning environment using both classroom computers (COWs) and a laboratory setting. They support of the teaching of mathematical concepts, integrating multimedia and technology to allow the exploration of topics and the application of mathematical skills are designed as part of the activities. Students are encouraged to interpret data using spreadsheets and graphing calculators, learn to use specific applications, and write results which are discussed and shared with group members and the class. Students are encouraged to acquire their own data, add additional examples which confirm ideas, and model the data to confirm ideas. Students are expected to select appropriate tools and use the tools to find values, make generalizations, and apply mathematical concepts.

The Future of Multimedia:

The use of multimedia in the mathematics classroom will grow as better materials are produced in response to the AMATYC and other national standards efforts impact the teaching of mathematics. Many projects are now producing the materials to support the instructor in revitalizing the teaching of mathematics and fully utilize the technology available. The MCP project, Fostering Multimedia Instruction in Mathematics[4], for example, is redefining teaching methods through the use of multimedia technology and has goals including: Transfer of Knowledge, Motivation, Retention, and Synthesis. PREMO is an acronym for Presentation Environment for Multimedia Objects[5] and promises the standardization of programming environments for the presentation of multimedia data. The presentation environment will support the "application oriented modeling and interaction at several levels of abstraction, .., still and moving computer graphics (animation), synthetic graphics, audio, text, still images, moving images (video), images from imaging operations, and other media types," and include "easy configurability, adaptability, and extendibility of standardized components."

It is easy to predict the ease of implementation of multimedia in the mathematics classroom, the integration of other disciplines, and available supporting materials will continue to improve. The technology is constantly evolving and as we continuously seek better ways to teach with technology and technical skill within the context of a mathematical topic.

4[Fostering Multimedia Instruction in Mathematics; Lugo G, Herman R; Mathematical Sciences, UNCW; Available [online] at: http://cte.uncwil.edu/et/articles/lugohrmn/index.html;10/30/96]
5[Presentation Environment for Multimedia Objects; Available [online] at: http://www.cwi.nl/Premo/premo.html;10/29/96]

Appendices

A. Bibliography

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2 Margarita/Salinelli; *Multimedia Mathematics*;Available [online] at: http://www.lib.siu.edu/itvc96/papers/5/; 10/29/96

3 Classroom Connect; *Citing Internet Resources*; March 96

4 Lugo G, Herman R; *Fostering Multimedia Instruction in Mathematics;* Mathematical Sciences, UNCW; Available [online] at: http://cte.uncwil.edu/et/articles/lugohrmn/index.html;10/30/96

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9 Multimedia in Mathematics Classrooms [online]; 26 Apr 96 http://acorn.educ.nottingham.ac.uk/ShellCent/1Tech/Multimedia.html

10 Assessments of Multimedia Technology in Education: Bibliography; [online]; compiled by Kotlas C; MSLS, Institute for Academic Tech; 9 Jul 96; Available at: http://www.iat.unc.edu/guides/irg-ll.html

11 Stull A, Wright E; *Life on the Internet: Mathematics*, A Student Guide; Prentice Hall; 1996

B. Examples and Ideas

Typically, the professor will draw representations on the board and hope the student "sees" what the instructor conceptualizes. Take the example of the cycloid - we hope to "convince the class that a point on th rim of a wheel will trace out a particular path." In multimedia presentation, "we show an actual wheel rolling across the screen. This motion can be repeated over and over giving the class the opportunity to describe the path traced by the moving point."[4] The use of multimedia or technological skills are required in the lab ideas included.

Math 112 Statistics: Excel Assignment

Data Set I

You are going to conduct a study of recent college graduates, comparing the starting salaries of a group of individuals hired into the education, communication, and sales job markets. A sample of twenty individuals from each of the job markets is gathered at random. The following data was obtained :

Individual	Education	Communication	Sales
1	25,634	28,518	17,345
2	21,654	26,121	21,645
3	24,745	27,034	23,543
4	23,656	25,934	19,345
5	22,678	28,334	26,563
6	25,456	23,356	25,435
7	23,466	25,344	20,533
8	23,502	27,854	24,743
9	21,456	26,863	26,444
10	23,875	27,345	22,546
11	25,754	21,823	27,349
12	24,674	28,333	22,132
13	22,564	25,345	27,943
14	21,455	27,932	20,235
15	24,543	25,657	19,453
16	23,124	26,700	28,120
17	19,456	27,980	27,459
18	20,456	24,753	21,550
19	21,344	25,451	28,453
20	23,455	23,890	27,457

1. Perform each of the following computations and print out the results:
    a. Round the values of this data set to the nearest $1000. Place the rounded salaries in columns of a spreadsheet. Label each entry appropriately.
    b. Make a histogram for the rounded salaries of each job market (three) and label the histograms.
    c. What is the mean (average) of each group? Median? Mode? Which group has the highest starting salaries, on the average? The lowest?
    d. Find the standard deviation of each group (use the function button). Which group has the greatest spread of salaries? The least?
    e. Make a line graph to compare the three groups. Use a different line symbol (or color) for each. Write a statement which explains the

comparison you see.

2. Choose an individual (one score) from each of the three groups. Compute the z-score of the individual, using the mean and standard deviation of the group the individual was taken from.
   a. Draw a sketch of the normal bell curve, placing the group mean and deviation scores in the appropriate positions. Put the individual selected on the curve, drawing a line where that individual is positioned in the group. You will have one sketch of each group, with the individual's z-score (three sketches altogether).
   b. Find the percentile for the individual's z-score and write this on the sketch.
3. If high salary were your only concern, which job market should you set your goal for? **Justify your answer**. Use any appropriate statistics in your justification.

Data Set II

A certain college mathematics professor (who shall remain nameless) has students maintain records on their study time and then compares their average nightly study time to the scores received on an exam. A random sample of the students showed the following comparisons:

Study Time (nearest 5 min)	Exam Score
20	62
30	75
55	83
25	72
35	87
45	85
20	68
30	77
35	74
50	95
55	89
40	80
35	82
40	92

1. Make a scatterplot of the data.
2. Predict the approximate number of minutes per night you must study in order to obtain a minimum grade of 70%.

Exploration of Functions

You are going to explore the concept of function in the following activity. Write complete descriptions and draw sketches for clarification, where appropriate. Don't forget you may need to set your graphing window parameters to different settings to see the graph well. Use the zoom and trace key functions to view the graphs better.

A. Graph each of the following equations on the TI-82 calculator and write a sentence to describe the shape of each graph.

1. $y = 3x - 7$ 2. $y = 2x^2 - 4$

3. $y = 2x^3 - 6x^2 - x + 6$ 4. $y = x^4 - 7x^2 - x + 6$

B. Graph the equations:

$y = x^4 - 7x^2 - x + 6$ How are these two graphs related?

and $y = -x^4 + 7x^2 + x - 6$

If two variables, x and y are so related that whenever a value is assigned to x there is automatically assigned by some rule of correspondence, a value to y, then we say y is a "function"of x.

C. Linear Functions: A function of the form $f(x) = ax + b$ where a and b are constants is called a linear function. The graph of a linear function is a line. Each line is unique in the slope (slant) and y-intercept (where it crosses the y-axis).

1. Investigate the changes in the graph of a linear function by varying the coefficient a. Describe how the line changes as you change the value of a.

2. Investigate the changes in the graph of a linear function by varying the constant b. Describe how the line changes as you change the value of b.

D. Quadratic Functions: A function of the form $f(x) = ax^2 + bx + c$ where a, b, and c are constants is called a quadratic function. The graph of a quadratic function is called a parabola. The parabola is symmetrical about a vertical line called the axis of symmetry, which passes through the point where the graph changes direction, called the vertex.

1. Investigate the changes in the graph of a quadratic function as you vary the coefficient a. Describe the changes in the parabola as you change the value of a. Change only the value of a, not b and c. For example, graph the functions:

   $f(x) = x^2$ (b = 0, c = 0) $f(x) = 2x^2$ $f(x) = -2x^2$

   $f(x) = x^2 - 2x + 1$ $f(x) = 2x^2 - 2x + 1$ and so on.

   Is there a relationship between the position of the parabola and the value of a?

2. Now investigate the changes made in the graph of the quadratic function when the value of b is varied. Describe the changes in the graph as it relates to the changes in b.

   Is there a relationship between the value of b and the equation of the line of symmetry?

   To test your conclusions, graph the quadratic functions: $x^2 - 2x$, $x^2 - 4x$, $x^2 - 8x$

   How can you find the axis of symmetry from the values of a and b?

3. The trajectory of a baseball after it leaves the bat can be described by the equation $h(x) = -0.05x^2 + 5.4x$ where h(x) denotes the height of the ball when it has traveled x yards from home plate. Use your graphing calculator to approximate:
   a. the greatest height reached by the baseball
   b. the distance the baseball is from home plate when it is at the point of greatest height
   c. the distance the ball traveled from the moment hit to the moment it falls on the ground

   Sketch the path of the ball.

E. Exponential Functions: When the rate of growth of a particular variable increases in proportion to the current value of a variable, we say the growth is exponential. For example, the rate of world population growth has become greater (and the graph grows steeper) as the population grows. The approximate world population, y, is expressed as a function of the year, x, by the following equation:

$$y = 10^{0.00389x + 2}$$

To provide a mathematical model for exponential growth, we use an exponential function of x, defined by an equation of the form:

$y = b^{Ax + C}$ where A and C are constants, and b is a postitive constant

When the constant A is negative, the exponential function describes what is known as exponential decay. This is when the rate of decrease is proportional to its current value. For example, when a radioactive substance decays, the amount of mass decreases as it emits radioactive particles. It is true in such cases that the time required for 50% of the substance to decay is called the half-life of the substance.

1. Using the above equation for world population, graph the equation and use the equation to determine the years in which the population reached 1 billion.
2. Use the equation to determine the years in which the population reached 3, 4, and 5 billion. Put this information in a table.
   Predict the year in which the world population will reach six billion.
3. Carbon 14 is an isotope of carbon found in plant matter which has absorbed it from the atmosphere. Carbon 14 decays exponentially from the time it is absorbed, allowing us to use the half-life to date materials. The percentage, y, of carbon 14 in the plant after x years is given by the following equation: $y = 10^{2-0.00005235x}$
   Graph the equation and determine the half-life of carbon 14. (Hint: find the value for x for which y = 50, using the trace function on the calculator.) Draw a sketch of the function.

Applications of Functions

The following data sets are appropriate to use in mathematical modeling. Use graphing and curve fitting to produce an appropriate graph and symbolic model for each.

A. World Population

Year	Population (in millions)
1650	550
1750	725
1850	1175
1900	1600
1950	2565
1990	5333

B. Minimum Distance from Sun to Planets

Planet	Minimum Distance (millions of miles)
Mercury	28
Venus	67
Earth	91
Mars	129
Jupiter	461
Saturn	838
Uranus	1669
Neptune	2760
Pluto	2756

C. 1990 Federal Estate and Gift Tax

Tax Computed On	Tax
20,000	3,800
40,000	8,200
60,000	13,000
80,000	18,200
100,000	23,800

Decimal Investigations: Patterns

Answer each of the following questions and find the requested values using a calculator. Give complete explanations and do not round values unless asked to do so.

1. Many patterns exist in the relationships between fractions and decimals. The first observations you will make are for decimal values which are called terminating decimals, which end or divide evenly.
   Find the prime factorization of the denominator of each of the following fractions:
   1/2 1/5 1/10 1/20 1/25 1/200

   Divide each fraction above (numerator divided by the denominator) to find the equivalent decimal value. Write them below.

   What do you notice about the prime factorization of the denominator and the resulting decimal? Can you find a specific pattern to the results? What is it?
   Find three more fractions which follow the same pattern you have identified.

2. Try the above investigations with fraction which have numerators other than one. Does the pattern still hold true or does it change?
   Give three examples.

   Try the previous investigation for improper fractions. Is your generalization still true? Write two examples below and a statement of the general pattern for terminating decimals.

3. The second type of fraction you will investigate is the repeating decimal. Find the prime factorization of the denominator of each of the following fractions.
   1/3 1/6 1/9 1/12 1/18 1/24
   Can you guess what the pattern of the decimal might be?

   Write the decimal form of each fraction below. Use the repeating decimal bar where needed (do not round values).

   What do you notice about the prime factorization of the denominator and the resulting decimal form? Is there a specific pattern you can identify?
   Find three more examples which follow the pattern you have found.

4. Investigate the pattern found in #3 above with fractions which have numerators other than one. Give some examples and write a general statement of the pattern you have found.

5. Some other fractions have interesting patterns. Investigate the decimal form of the following fractions and give a statement of the pattern you have found.
   1/9 2/9 3/9 4/9 5/9 6/9
   7/9 8/9 How many did you check?

   What would 9/9 equal (if you follow the pattern)?
   Do you have questions regarding this value?

   Another interesting pattern is generated from division by 7. Starting with 1/7, investigate this pattern in the same way you did the previous one. How many sevenths did you have to look at to see the pattern? What is the pattern you see?

   What about the other fractions that have denominators which contain factors of 7? Write a statement of the pattern for the decimal form of fractions with denominators containing a factor 7.

Getting Started with Minitab

The following information is designed to get you started working with the statistics application Minitab, which is available for use in the General Purpose Computing Lab. The information is to familiarize you with the page format and basic commands in order to easily work with Minitab.

General Purpose Lab Instructions

Minitab is available for student use in the General Purpose Computing Lab, during posted hours. Each student must bring a 3 1/2 HD disk for saving work, although it may have other saved IBM compatible files. You must log in and out of the lab, as well as follow other directions as specified by lab personnel.

Minitab is available in the Math software menu, which you can open in Windows95 in either of the following ways. First, you can open the start menu at the lower left corner of the screen by clicking on the Start button, moving the mouse cursor up to programs - right to math software - right to minitab and click to open. Otherwise, move the mouse cursor to the program icons in the desktop screen (top or right in Windows95, notice the purpose of the button pops up) until you highlight the math software button, click on the button to open the math software file and click on the Minitab icon to open the program. If you need assistance, ask your instructor or the lab assistant.

The Minitab Worksheet

The main Minitab window has four subwindows: the Session Window, the Data window, the Info Window, and the History window. You can position and size these windows any way you wish. Across the top of Minitab is the menu bar from which you can open menus and choose the basic commands for files, editing, calculating, and graphing. Notice the on-line help menu to the right of this line.

The session space provided in Minitab (top window) is where you enter and work with the statistical calculation and basic commands. It displays non-graphical output such as tables of statistics and model equations. The Data window is where you enter, edit, and view your data. The Info window summarizes the worksheet and the History window records all commands. To move any window, point to the title of the window with your mouse, hold the left mouse button down, and drag the window to the desired spot, then release the mouse button. To bring a partially hidden window to the front, click anywhere in the window.

The worksheet consists of columns, constants, and matrices. Most of your work will be done with columns, which you can view in the Data window. Minitab provides many ways for you to enter data: type it, paste it, open it from a file, or generate it. Notice the arrow in the upper left corner of the Data window. It changes the direction of data entry. To highlight an entire row, click the row number and to highlight an entire column, click the column heading. To change the width of a column, point to the top of the line dividing the columns until the cursor becomes a two-sided arrow. Press the mouse button an drag the line to the desired width.

Remember to save your data frequently! Choose File - Save Worksheet (As) from the main menu to save it. Once you have entered all the data, make sure you have saved it so you only have to open it when beginning a session. To copy data to the clipboard, highlight the data, then choose Edit - Copy Cells. You can copy from another package, such as Lotus or Excel, another Minitab worksheet, or the session window. Once you have copied the cells on the clipboard, you can paste them where you want to display them. Use open worksheet to *replace* the data in the current worksheet with data from a file. Choose Merge Worksheet to *add* data from a file to the current worksheet.

Generate Patterned Data

To generate data, use the Calc commands to create new values. Choose Calc - Set Patterned Data, and complete the dialog box, clicking OK when you have set the data pattern. Remember to store the result in a free column, which can be named. Choose Window - Data to view the data in a new column.

Generate Random Data

To generate random data, choose Calc - Random Data - Normal for generating random numbers of a distribution with a selected mean and standard deviation. Complete the dialog box with the desired number of rows and columns to store the values in. Choose Window - Data to view the generated data.

Analyze Your Data

Minitab contains sample data sets, which can be used to learn the basic commands in an analysis. Consider the sample data set stored in the file PULSE.MTW, for example. Students in an introductory statistics course participated in a simple experiment. Each student recorded his or her height, weight, gender, smoking preference, usual activity level, and resting pulse. Then they all flipped coins, and those whose coins came up heads ran in place for one minute. Then the entire class recorded their pulses once more.

Choose File - Open Worksheet, click drives and directories until you are in the \MTBWIN\DATA directory. Scroll down the list of files until you see PULSE.MTW, then click to highlight the file name. Click OK to open the file, and Proceed, if you see a message box asking whether to proceed with Retrieve.

Basic Descriptive Statistics: Choose Stat - Basic Statistics - Descriptive Statistics. A window will open with a variable list, which you will select the desired variables from. Highlight the variable(s) and click select. When selection is completed, click OK. For example, when you select Weight from this sample file, Minitab displays output in the session window; see the count, means, medians, etc. displayed.

You can perform other analysis functions under the Stat menu in the same fashion. You may also manipulate the data by choosing the Calc - Mathematical Expressions and type the desired manipulation of columns under the expression title in the dialog box.

Graph Your Data

To graph your data (there are *lots* of ways to display graphs) choose from the graph options on the main menu. To draw a scatterplot of two variables, choose Graph - Plot. You will see a dialog box with a variable list on the left to select from, as before and graph variable options. In the Y box, select the desired variable for the vertical axis and for the X box, select the horizontal axis. To display a different symbol for each group, click the down arrow beside For each and click Group; click in the box labeled Group variables and select how your want them grouped. Click Edit Attributes to change the plot symbols by type, color and size. Click OK when done. To add a title to the graph, click the down arrow in the plot box beside Annotation, click Title and type the title desired.

Save and Print Your Work

To save your work, choose File - Save Worksheet (As) and name the file something easy for you to remember. To print your work, click in the data window, or other desired window to print, then choose File - Print Window. To print part of the data window, highlight the part you want to print before choosing File - Print Window and it will print the selected data. To save a graph, click in the graph window you want to save, then choose File - Save Window As.

C. Interesting Sites and Applications to Visit

Mathematics Archives; Available at http://archives.math.utk.edu/cgi-bin/interactive.html

McGraw Hill Encyclopedia of Science and Technology, 7th Ed.; [online] http://sulacco.library.ucg.ie/services/elec/mcgraw.html

Effective Teaching: Cybrary; [online] accessible resources: data, facts, images, sounds, freeware and shareware; Available at: http://cte.uncwil.edu/et/cybrary.htm#data

Philadelphia Graphing Calculating Committee; [online] links to collections of programs for the TI calculators and mathematics web pages; Available at: http://forum.swarthmore.edu/~stevek

Eisenhower National Clearinghouse; [online] Available at: http://www.enc.org

American Mathematical Association of Two-Year Colleges; http://www.amatyc.org/amatyc

National Council of Teachers of Mathematics (NCTM); http://www.nctm.org/nctm

Mathematical Association of America (MAA); http://www.maa.org/maa

History of Mathematics Home Page; http://aleph0.clarku.edu/~djoyce/mathhist/mathhist.html

Calculus, Concepts, Computers, and Cooperative Learning; http://www.math.purdue.edu/~ccc/

Mathematical Animation Gallery; http://mathserv.math.sfu.ca/Animations/animations.html
On-line Mathematics Dictionary; http://www.mathpro.com/math/glossary/glossary.html

TI Graphing Calculators; http://www.ti.com/calc/docs/graph.htm or http://www.ti.com/calc/index.html

Image Processing for Teaching; http://ipt.lpl.arizona.edu

D. Student Comments

A. Do you think multimedia helped you learn the objectives of the course: did it increase your understanding and support your learning? Or was it a needless distraction?

Yes, I like and understand things better when I can get hands-on activities.

It was helpful to have access to multimedia. I find the class more interesting when you have the chance to observe multimedia than when it is strictly lecture.

Multimedia increased my understanding about the different ways and uses of the computer,digital camera etc. I don't feel that it was a needless distraction.

Yes it did increase my understanding of mathematics and kept my interest more than lecture & note taking.

Multimedia helped me learn objectives and helped me understand what was being taught better.

The multimedia tools definitely enhanced my learning and conceptualization of mathematics.

I found it increased understanding of multimedia and supported the learning about the subject. I think it's going to be a great teaching\learning tool because of the versatility it will offer to teachers and students.

It was a Godsend and helped us to learn more complex problems.

Yes it helped support learning and it made it more fun. It also helped show us what's available for us to use when we teach.

I know that multimedia helped me learn the objectives because it was presented very well and was also interesting.

B. Do you like using calculators and computers?

Yes, great visuals.

I like using the TI-82 and computers very much. I look forward to being able to purchase my own TI-82. I use what I learn on the computer here and apply it to my computer at work.

Once I figured out the Mac (power mac) I enjoyed using it. My TI-82 is invaluable for me in other classes besides math.

I really liked using them both, especially the computer. Computers are so much a part of every busi-ness it is very important to know how to use one.

Yes, math without them would not be as much fun.

Yes. I found using the calculators easier than the computers.

I like using the calculators and computers, it made me want to explore and experiment.

Yes, I think that it's an important part of learning and teaching.

C. Was it better learning; did it increase your ability to visualize and think mathematically?

Using the calculator and computer helped me actually see what was going on. I'm a visual learner and need to "see" what is being taught.

Using the technology made the subject more interesting.

It was better learning since I am a visual learner and need to see using hands-on methods.

The multimedia helped me think more mathematically.

Yes some activities were easier than others, good to work in groups because not all of us are familiar with computers.

All of the multimedia tools have helped me because I am a visual learner. I wish time allowed for emphases on some of the aids.

Yes, I found using graphics and other visuals made different math subjects a lot more interesting and easier to understand.

It was better learning--the technology made things clearer and easier to understand. Mathematical learning is much easier when you have a picture to show you what you are doing.

For me, to see something is learned easier than to learn it out of a book. Just a little difficult changing my trained thought of learning.

It was good to see the results, graphs, and equations on the calculator. It reinforced my ability to visualize and to think about my results.

D.Other comments.

This class was wonderful, interesting & more relaxing Ha Ha. Good job Lynn.

I really enjoyed getting to see and use the laser disc. I think it is very important for teachers to know how to operate the different types of equipment.

There should be more multimedia in the class room.

It was better learning since I am a visual learner and need to see concepts using hands-on methods.

Needed more time to work on certain activities. There should be outside help available other than instructor. She should have an aide like geology professor has. that really would help us get the extra help we need.

I've had a TI-82 for 2 years and am still learning on it, but it has been invaluable to me since I first started using it. Now, I'm looking forward to working on the TI-92 someday soon.

I would definitely recommend keeping the technology in the class because the students need a chance to see what's out there and what they have available to them to use and they need a chance to use that technology and see how it works.

Lynn, You've done good in presenting this stuff considering it's all so new to you.

I really enjoyed this aspect of the class, I feel fortunate that I was able to learn about statistical applications (spreadsheeting), that are essential to teachers. All the software that was presented was geared toward students learning and how to approach it toward the student. I only wish that we had more time in class to work on these applications. It was hard to work around my schedule.