Bring Functions and Graphs to Life with the CBL

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AMTCYC's Crossroads in Mathematics includes function as one of its Standards of Content. This paper describes difficulties students experience with the function concept, studies conducted to determine the effect of technology on mathematics education, and the potential effect of the CBL and graphing calculators on students' learning of functions and graphs.

Introduction

It can be argued that the function concept is the single most important concept from kindergarten to graduate school (Harel and Dubinsky, 1992). The function concept is a central one in mathematics which grows in importance as one progresses in the depth and breadth of one's understanding of mathematics (Yerushalmy and Schwartz, 1993). Yerushalmy and Schwartz (1993) believe that the function is the fundamental object of algebra. The American Mathematical Association of Two-Year Colleges [AMATYC] (1995) in its Standards document includes function as one of its Standards of Content and states, "Students will demonstrate understanding of the concept of function by several means (verbally, numerically, graphically, and symbolically) and incorporate it as a central theme into their use of mathematics."

Introduction of Functions and Graphs

Functions and graphs generally do not appear until the upper elementary grades or later (Leinhardt, Zaslavsky, and Stein, 1990). The introduction of functions and graphs is a critical moment in mathematics education since it presents a setting with the opportunity for powerful learning to take place and since the concepts of functions and graphs are fundamental to more sophisticated parts of mathematics. Functions and graphs represent one of the earliest points in mathematics at which students use one symbolic system to expand and understand another. A function can be described by a verbal or written statement, by an algebraic formula, as a table of input-output values, or as a graph (National Council of Teachers of Mathematics [NCTM], 1989).

Algebraic and graphical representations are used to jointly construct and define the mathematical concept of function; consequently, functions and graphs cannot be treated as isolated concepts (Leinhardt et al., 1990). Since functions and graphs are two symbolic systems used to illustrate each other, demands are placed on the learner in terms of new ideas, notational uniqueness, and symbolic correspondences.

Students' Difficulties with Functions and Graphs

Students experience a major leap in their mathematical development when they are introduced to the concept of the graph of a function in two variables (Herscovics, 1989). Results from the Second Mathematics Assessment of the National Assessment of

Educational Progress (Carpenter, Corbitt, Kepner, Linquist, and Reys, 1981) indicate that the majority of students do not manage this leap. Carpenter et al. (1981) found that students could graph ordered pairs of numbers in the Cartesian plane but that most did not understand the relationship between equations and their graphs.

Results from the Fourth National Assessment of Educational Progress (Brown et al., 1988; Silver et al., 1988) indicated that U.S. students had a limited understanding of function concepts and of graphing. Brown et al. (1988) reported that secondary students demonstrated some intuitive knowledge of functions but experienced difficulty with items covering a broad range of function concepts.

Kenelly (1986) contends that calculus students experience difficulty with function concepts. He surmises that beginning algebra students fail to form a conceptual understanding of variables; as a result, for many students variables are simply symbols used in manipulative practice exercises and functions are "ordered pairs of these things." Students miss the idea that functions capture the spirit and essence of connections and interdependencies, and they fail to see that functions embrace the elements of input and output, control and observation, and cause and effect. Epps (1986) says that beginning calculus students do not know the abstract definition of graph of a function. They can plot and connect points, but they do not know that for a function f, f(x) is the height to the graph of f at x.

The Potential of Technology

Technology has the potential to enhance the understanding of functions and graphs, but this potential has not been fully realized. The hand-held scientific calculator was introduced in the 1970s, but McConnell (1988) states that even though calculators are found in every nook and cranny of the United States, many mathematics students are still subjected to topics that assume a world with no calculators and are forbidden to use calculators in their classrooms. The microcomputer has not had the impact on the teaching and learning of mathematics that had been predicted (Barrett and Goebel, 1990) since many schools do not have a computer in each mathematics classroom and since many educators have trouble defining the role of the computer in the classroom.

Most algebra teachers indicate that they use computers for demonstration purposes only (Demana and Waits, 1992). The computer has had a great impact on mathematics, but mathematics is still being taught in most college courses just as it was 30 years ago as a paper-and-pencil discipline (National Research Council, 1991). Demana and Waits (1992) are convinced that if teachers and students rely solely on desktop personal computers, no meaningful reform will occur in mathematics education in the 1990s; students must use computers on a regular basis for both in-class work and for homework if significant changes are to be made in the mathematics that students learn in the 1990s. Demana and Waits advocate the use of inexpensive pocket computers (graphing calculators) in mathematics education.

Computer- and calculator-based graphing has the potential to change the way mathematics is taught and learned (Waits and Demana, 1988) since this technology enables students to handle more complicated, realistic, and noncontrived applications. The graphing calculator can transform the mathematics classroom into a laboratory environment where students use technology to investigate, conjecture, and verify their findings (NCTM, 1989). Graphing calculators have the greatest potential of all of technological innovations to impact on the teaching of precalculus and calculus (Dion, 1990).

Effect of Technology on Mathematics Education

Several studies have yielded inconclusive evidence of the value of the graphing calculator in mathematics education. Rich (1990/1991) conducted a study to examine the effects of graphing calculators on precalculus students' achievements, attitudes, and problem-solving approaches in which two classes were taught precalculus using CASIO fx-7000G graphing calculators and six classes were taught precalculus without graphing calculators. The study did not provide evidence of an overall achievement effect of graphing calculator instruction. Rich concluded that students taught precalculus with the use of graphing calculators acquire a better understanding of the relationship between an algebraic equation and its graph.

Estes (1990) conducted a study to investigate the effects of implementing graphing calculators and computer technologies as teaching tools in Applied Calculus in which she designed and taught two experimental classes in Applied Calculus with the use of CASIO fx-7000G graphing calculators and a Macintosh SE computer with overhead projector capabilities. Special assignments were developed and used to facilitate learning the concepts of Applied Calculus with a graphing calculator. Students in a control group were allowed to use calculators, but did not receive any special attention to effective uses of the graphing calculator to facilitate conceptual learning. On end-of-semester tests, the experimental group scored significantly higher than the control group on conceptual measures. No significant difference was found between the groups on procedural measures.

Caldwell (1994/1995) conducted a study to determine the effect of the TI-81 Graphics Calculator as a learning tool on college students' understanding of concepts and performance of procedures involving functions and graphs. Two college algebra classes, the treatment group, studied functions and graphs with the use of TI-81 Graphics Calculators. Two college algebra classes, the control group, studied functions and graphs with the use of scientific calculators without graphing capabilities. At the conclusion of units on functions and graphs, a concepts test and a procedures test were administered to both treatment and control groups. The treatment group scored significantly higher than the control group on the procedures test. There was no significant difference between the two groups on the concepts test.

Graphing calculators provide opportunities for students to connect graphical images, symbolic expressions, and sets of related numerical values (Caldwell, 1994/1995). With graphing calculators such as the TI-82 and TI-83, students can represent functions numerically as tables of input-output pairs, symbolically as algebraic representations, and graphically as plots of input-output points. Students need to translate across these representations and connect these representations to physical contexts. A device which can aid students in connecting numerical, symbolic, and graphical representations of functions to physical contexts is the Texas Instruments Calculator-Based Laboratory System (CBL).

The Calculator-Based Laboratory System

With the use of a CBL and a graphing calculator, students are able to physically observe a functional relationship between two variables and then observe a graphical representation of the relationship as a scatter plot. Students may also observe a numerical representation of the relationship by tracing among the points of the scatter plot and by scanning the lists in which data are stored. Students may then use the graphing calculator to determine an equation for a curve to be fitted to the data and thus obtain a symbolic representation of the relationship.

The CBL is a portable, handheld, battery-operated data collection device for collecting real-world data (Texas Instruments Incorporated, 1994). With the CBL and appropriate probes or sensors, one can measure motion, force, temperature, light, sound, and more. Data can then be transferred directly to TI-82, TI-83, TI-85, and TI-92 graphing calculators for analysis. The data is stored in lists in the graphing calculator, and data points are plotted as a scatter plot on the calculator screen. Several books have been published with step-by-step mathematics and science experiments for the CBL including programs for the experiments that can be downloaded into graphing calculators. A few of the experiments which have been used by the author are described in this paper.

CBL Mathematics Experiments

Using the HIKER program with a CBL, a graphing calculator, and a motion detector, students walk in such a way that the graphs of their distances from the motion detector with respect to time form different shapes. Experiments with the HIKER program allow students to develop the concept of slope. Walking away from the motion detector at a constant rate generates a line with a positive slope while walking toward the motion detector at a constant rate generates a line with a negative slope. By varying one's rate during a single walk or by walking away from and toward the motion detector during a single walk one may generate graphs of many different shapes. These graphs represent functions, and by tracing among the points on the graphs, one may determine domains, ranges, and extrema of functions. Experiments using the HIKER program provide excellent opportunities for students to develop and refine such concepts as function as a correspondence; function as a set of ordered pairs; domain and range of a function; relative maxima and minima; absolute maxima and minima; and increasing, decreasing, and constant functions.

Several CBL programs allow students to work with specific types of functions. Using the BALLDROP program with a CBL, a graphing calculator, and a motion detector, one may drop an object from directly over the motion detector or toss an object and let it fall directly over the motion detector. A plot of the collected data includes points before the object is dropped and after it is caught. These points may be removed with the SELECT program, leaving only the points representing the times and distances during the fall of the object. The plot of the remaining data will appear to be quadratic. By tracing among the points on the plot, one may approximate the velocity of the falling object at different times, and this can lead to a discussion of limits. One may select STATCALC QuadReg L3,L4 on the graphing calculator to determine the coefficients a, b, and c of a quadratic function of the form y = ax2 + bx + c to model the relationship between the times and distances of the falling object. The equation of the quadratic function may be pasted into the Y= list and then plotted on the calculator screen along with the scatter plot of the collected data.

Using the HOOK program with a CBL, a graphing calculator, a motion detector, and a slinky, one may observe an example of harmonic motion. The plot of the resulting data can be used to illustrate the concepts of periodic function, amplitude, period, phase shift, and vertical translation. One may use the graph of the data points to find the values of A, B, C, and D to fit an equation of the form $y = A\cos(Bx + C) + D$ to the graph. By using SinReg on the TI-83, one may determine the equation of a sine curve to fit the plotted data. The Force1 program can be used with a CBL, a graphing calculator, a force sensor, and a slinky to obtain a plot of harmonic motion. The Sound program can be used with a CBL, a graphing calculator, a microphone, and a tuning fork to illustrate periodic functions.

Using the LIGHT program with a CBL, a graphing calculator, a light sensor, a light source, and a meter stick, one may investigate the inverse square law. The light sensor is placed at several different distances from the light source, the intensity of the light at each distance is determined by the CBL, and the resulting plot of distance versus intensity is shown on the calculator screen. By tracing among the points of the plot one may approximate rates of change of intensity with respect to distance. Using STATCALC PwrReg L2,L4 one can use the graphing calculator to determine the equation of a power function to fit the plotted data.

The HEAT program can be used with a CBL, a graphing calculator, a temperature probe, and different pieces of apparatus in the study of exponential functions. With the HEAT program, the CBL collects 36 readings of time and temperature. The time in seconds between the readings is set by the person conducting the experiment. The resulting data points are plotted on the calculator screen. Time data is stored in L3, and temperature data is stored in L4.

An experiment can be conducted by heating a cup of water, removing the heat source, placing the temperature probe in the water, and then letting the CBL collect data over a period of several minutes. One may trace among the data points and approximate the rate of change of temperature at different times. One may then select STATCALC ExpReg L3,L4 to obtain the coefficients a and b for an exponential function of the form y = abx to model the relationship between the time and temperature of a cooling object. By using natural logarithms, one can determine a corresponding exponential function with base e. The equation of the exponential function can be pasted into the Y= list and plotted on the calculator screen along with the scatter plot of the data.

An experiment requiring less time can be conducted by folding a piece of aluminum foil, placing the temperature probe between the layers of the foil, flattening the foil tightly around the probe, and then heating the foil with a hair dryer. When the foil is hot, the hair dryer is turned off and the HEAT program is started. Data points are collected by the CBL and plotted on the calculator screen every few seconds as the foil cools. An exponential function can then be determined to model the data. An experiment requiring even less time can be conducted by placing the temperature probe in a cup of hot water for a short period of time and then removing it and placing it in a cup of ice water, starting the data collection with the HEAT program as the temperature probe is removed from the hot water. An exponential function can also be determined to model this data. The temperature probe can also be placed in a cup of ice water and then removed and placed in a cup of hot water generating a heating curve. By selecting STATCALC LnReg L3,L4 one can use the graphing calculator to determine a logarithmic function to model the data.

Conclusions

One of the basic principles of the AMATYC Standards is that mathematics must be taught as a laboratory discipline (AMATYC, 1995). There must be an emphasis on effective mathematics instruction involving active student participation and in-depth projects employing genuine data to promote students' learning through guided hands-on investigations. The use of the CBL along with the graphing calculator promotes this type of learning environment.

With only one CBL in the classroom, students can physically observe a functional relationship between two variables while data is being collected by the CBL. Data can then be transmitted from the graphing calculator used with the CBL to students' graphing calculators. Students can then analyze the numerical representation of the data by scanning the lists in which the data are stored. Students can plot the data on their calculator screens to obtain a graphical representation of the data. By analyzing the plotted data points, students can determine the type of function that the points seem to fit and then use the graphing calculator to create an equation for the curve of best fit to obtain a symbolic representation of the data. Students can then plot the curve of best fit on their calculator screens along with the scatter plot, and by using the equation of the curve of best fit, they can predict values of one variable when given values of the other variable.

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