AMATYC 32nd Annual Conference
Saturday Awards Breakfast Session
Keynote Speech:

"BLOWN AWAY: WHAT KNOT TO DO WHEN SAILING" BY SIR RANDOLPH BACON III

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Being a tale of adventure on the high seas involving great risk to the tale teller, and how an understanding of the mathematical theory of knots saved his bacon.

In this talk, Sir Randolph Bacon III tells of his adventures during the Cambridge-Oxford regatta on the Thames. These adventures involve a knot in his sheet, a shark, a hurricane and various other calamities. Ultimately, he must figure out whether the knot in his rope can be disentangled, while the ends are fastened down. He remembers some knot theory taught to him by Mel Slugbate:

How do you tell if two pictures of knots are really the same knot?

This was a question of interest to the German mathematician Kurt Reidemeister. He realized that certain moves you do to a picture of a knot preserve the knot.

These are called Type I , Type II, Type III Reidemeister moves.

Two pictures of knots represent the same knot if and only if you can get from one to the other by a sequence of Reidemeister moves.

So if we have an attribute of knot pictures that is preserved by Reidemeister moves, it becomes an attribute of the knot rather than of an individual picture.

So our problem is solved. Given a picture of a knot, all we have to do is check to see if there are any sequences of Reidemeister moves that turn the projection into the trivial projection. But how many moves might that be?

Hass and Lagarias proved that if we have a picture of the trivial knot with n crossings, it will take fewer than 2^100,000,000n Reidemeister moves to turn one into the other. So there is an upper bound.

Be that as it may, we have not yet found a means to tell if a knot is actually knotted. For that purpose, we will turn to

Tricoloration:

Color each strand in the picture with one of three colors.

Rules:

1. Must use at least two colors in the projection.
2. Each crossing has one or three colors meeting.

Tricoloration is preserved by Reidemeister moves.

Theorem. If one projection of knot K is tricolorable, they all are.

Example: So trefoil knot and trivial knot are distinct! We proved it!

Now what about our knot?

It is not tricolorable, so we do not know if it is nontrivial. Maybe it is and maybe it is not.

But Mel Slugbate said that you can generalize tricolorability to p-coloration, where p colors are represented by the p integers {0, 1, 2, …, p-1}. At each crossing, you must satisfy a+b = 2c(mod p) if a, b and c are the integer labels placed on the two undercrossing strands and the overcrossing strand respectively. P-coloration is also preserved by Reidemeister moves, and therefore only depends on the knot, rather than the picture of the knot.

So he attempted a 5-coloring but no good. He attempted a 7-coloring. Lo and behold, it worked. Since the trivial knot is not 7-colorable, but this knot was, he knew it was a nontrivial knot and he would not be able to disentangle it without freeing the end of the sheet from the shark. Sir Randolph Bacon takes matters into his own hands, frees the shark from the line, overcomes the greatest hurricane ever to hit the Atlantic, and survives his adventures.