Poker for the Mathematics Classroom

(Probability/Statistics)
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Abstract: This document represents a summary of the overheads used in this workshop at AMATYC San Diego, Saturday November 12, 2005, 3:45 - 5:45 pm. Poker and the deck of cards helped civilize the West. Patterns on cards appeal both to the intellect and to the eyes; they enable concrete human senses to inform abstract mathematics. The game Texas Hold'em is sweeping the nation--being efficient for industry and pleasing for human psychology. Use it to garner student engagement with important math skills and concepts. This document appended with related probability activities from other AMATYC workshops.

Outline of Workshop, 2005

I. Introduction
II. Humans and ....
    - Models (discuss)
    - Standard deck of cards:
        - evolution, structure, set theory
III. Poker, a natural game
    - History, structure, and
    - Commercialization (discuss, break)

IV. Industrialization of the game
    - Rules and probability
    - Game theory
    - Machine gaming, Texas Hold 'em
V. Mathematics in poker
    - Play, compete, compile, analyze, compare
VI. Closing statements

“Poker is a game of people.
... It’s not the hand I hold,
it’s the people I play with.”

Amarillo Slim

Playing card stenciling (detail from a French 17th century painting)

“Hold ‘em is to stud
what chess is to checkers.”

Johnny Moss

We humans do like to manipulate things and information in work and play. Given the logistics of living in a material world, we tabulate information about that which was manipulated.

Diverse people with common goals--to "figure out" the math--can benefit greatly if they cooperate. Try it in the math classroom. The safety of a cooperative classroom is the place to teach and practice the art of civil competition. Games with math manipulatives and chance provide skill and drill and hold attention.

Process in the sciences: 1st) Reality observed.
2nd) An intellectual description devised: ie. a theoretic model
(with its logical classifications)
3rd) Verification of theoretic model through continued observation, and by seeking anomalies (essential to science, considered optional in school?)

Watching the world unfold requires more patience than allowed by educational models. Society found a desired alternative mode of verification:

Manipulation ... of things to get at arithmetic, of numbers to get at algebra, of algebra to get at calculus ... of things to get at discrete methods, of models to get at design, of measures to get at sciences (and the "real" numbers) ... is a lot of our life!

Manipulatives in the mathematics classroom enable teaching with concrete experience, in a safe and collegial atmosphere, with live investigations and concept development, mathematical modeling, measurement, remediations and the meaning of division ... (time permitting?)

"MODELING" is a popular human activity

From Funk and Wagnall's Standard Dictionary, International Edition, 1959:

model (noun)
1. An object, usually in miniature, representing accurately something to be made or already existing; ... a plan or drawing: a model of a building
3. ... that which is taken as a pattern or an example.

model (verb transitive) 1. To plan or fashion after a model or pattern
(verb intransitive) 7. To assume the appearance of natural form

SIMULATION: **“modeling”**
1) manipulation of physical/symbolic model (defn noun 1)
2) to gain insight into the theoretic model (defn noun 3)
3) to gain insight into the reality

Results should lead to fine tuning/adapting the models to reflect reality more closely. Often used to demonstrate the models’ superiority over reality?

Group Activity: Discuss (after introducing yourselves) the notion of modeling in your mathematics classrooms, including difficulties. We have 8 minutes for this.

Responses included concerns for: Time both in use and in distribution among students, Class Size makes activities not practical, Technology as the preferred modeling device, Relevance often contrived, Availability requires financial support and advanced planning, Validity to course goals.

In teaching statistics and introductory probability, playing cards minimize these logistic difficulties.

So we now shift to look at my particular favorite model, the deck of cards, to demonstrate to you their ultimate applicability to our jobs teaching math to humans:

<u>Socio-economic theoretic model: *Theoretic partition of humanity into four ...*</u>

<u>*by Chinese CASTES, c. 1000 A.D.*</u> Born to one:

Peasant ***Military*** ***Professional*** ***Religious***

This model attempted to *well-define* human society by prescribing:
- each individual’s relation to each others’ (hierarchic)
- what you could do in your life (occupations, et al.)
- what you could wear, where (costume)

(And, by the same mathematical structure, if not social intent:

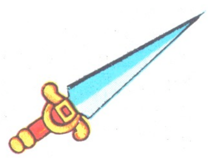
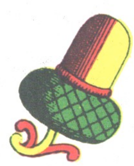
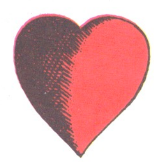
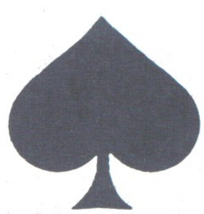
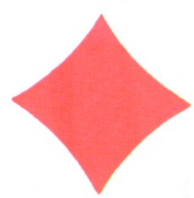
<u>*by US Gov’t SURVEY, 1998.*</u>

Check one: ***≤ $15,000*** ***≤ $45,000*** ***≤ $100,000*** ***> $100,000)***

By 1250 A.D. ... the Khans, Marco Polo, and world trade enabled social and cultural exchange among people! ...

The Chinese model spread and evolved.

The Physical Model: History of “Suits” and their Pips

	Peasant	Military	Professional	Religious
Chinese <1000 A.D.	 coins	**Strings of coins**	**Tens of strings of coins**	**Myriads of tens of strings of coins**
Persian/ Arabic <1200 A.D.	**Polo-sticks** جوكان ***Jawkân***	**Swords** سيوف ***Suyûf***	**Coins** دراهم ***Darâhim***	**Cups** طومان ***Tûmân***
Egyptian ≈1200	***sticks or wands***	***daggers or swords***	***gems or pentacles***	***chalices***
Spanish < 1400				
Germanic < 1500				
Stylized, standard < 1600				

Notice that the physical model uses color to add depth of reality: the most common social associations between castes/classes share the same pip color, Black or Red.

Breakdown of castes into 13 levels:

Ten levels per caste describe common people, assumed measurable / classifiable--both in kind and interaction; represented by numbers (1 through 10), given visually on common cards by the simple symmetries found on our standard deck.

Three levels (now, maybe four before Western influences) of which describe more unique people, represented only by their properties, with unique qualities guiding interaction with all other cards, where the symmetries are more intricate and similarities few. The kings wear white hair and beards, queens hold a flower, and knaves, also called jacks, wear golden hair and no beards.

Summary of "natural" classifications of cards (ie. equivalence classes):

2 colors, 26 cards in each; typically red and black.
4 suits, 13 cards in each; now spades, clubs, diamonds, hearts, indicated by pips.
13 kinds, 4 cards in each; one from each suit, labeled Ace, 2, 3, ... 10, J, Q, K.
2 types; 40 numeric and 12 "picture" or "face" cards.

Starting with the Renaissance in Southern Europe, as the Moors moved north, they brought new things to Europe: medicine, mathematics (especially accountants on merchant vessels), as well as playing cards and the game poker. All of these received condemnation from people in power at the time--secular and theologic.

Yet, knowledge of them all gained common popularity, rapidly. Playing cards and card games in particular have been condemned by kings, princes, potentates, Churches, Schools, and now the Taliban (see *Missoulian* article "Afghan officers arrest former Taliban minister" on 18 April 2003). Yet they remain popular even in the face of modern technology--solitaire on every computer! In October 2005 in the *Missoulian*, a guest editorial reminded us that major military leaders of American history (Washington, Grant, Eisenhower) all played cards; did gaming help improve their strategic thinking? Probably, in spite of Protestant protestations.

<u>Expansion-enabling technology for deck</u>

< 1000 A.D.   Chinese paper and printing
- Block printing on tiles, heavy paper
- Popular among royalty (invented by ? 969 A.D. ?)

< 1300   World trade routes
- Italy, Marco Polo, the Silk Road, the Turks
- Muslim expansion, Iberian peninsula mines

< 1400   Renaissance Art, rise of Leisure

< 1500   Johann Gutenberg's printing press (Cash for a starving artist!)

< 1800   New Orleans, Mississippi River trade, riverboat culture of movement
- Improved printing, cutting, stock
    - Rounded corners eased shuffling! (Activity: try with 1850s deck.)
    - Factory system kept price down. (See opening lithograph.)

1800s America moved west in wagons
- "Corner pips" printed for shortcut
- New! Blank card in each deck free.   <1870
- Players adopted blank as wild card   <1871
- Joker (from tarot Jester?) introduced   <1875

Improved deck as model of real society: The one of every bunch who fits nowhere and everywhere.!?

<u>Poker across continents and centuries</u>

? 969 A.D. Chinese emperor Mu Tsung & wife (inventress?)
- "domino cards" or "game of leaves"

< 15th Century   Persian game ***As ras***

15th Century   Regional variations similar in structure
- Italian game *Il Frusso*, later *Primiera*
- French game *LaPrime*, later *L'Amigu*, *L'Mesle* or *Brelan*
- English game *Post and Pair* and *Brag*
- German game *Pochen*

1718   Louisiana territory game *Poque*
- Five card hand from 20 card deck, one bet & showdown
  maximum of four players per game

1800   Faro and 3-card Monte still most popular card games on the circuit

1834   J. H. Green documented "the cheating game," **Poker**

≈ 1840   Full-deck Poker introduced
- Lowered typical hand; enabled more players per game
- The "flush" arose outside of royal setting, accepted

1860s **5-card "Draw"** poker introduced--a second bet
- 5-card "Stud" poker introduced--four bets possible
- "Straight" became an officially-valued hand
- "Bluffing" entered as strategy
- Faro and 3-card Monte lost favor on the circuit

[The complexity barrier thus broken through, new twists began being invented and accepted.]

1870 "Jacks or Better" introduced, also known as "Jack-pots"
- Added more bets, increased single pots, strategy

≤ 1875 The new Joker employed as a wild card
- Chance games became less predictable/determined

1900 "Low ball" and "split pot" games introduced
- Increased pot sizes, number of winners & 'confistications'

1930s "**7-card stud**" popularity rose above 5-card draw's
- Up to six bets possible; maximum of 7 players per deck
- No regrets for mis-chosen thrown cards.

(Better: fewer cards "down" makes cheating harder, no discards to watch.)

1931 Nevada legalized games of chance

1970 First World Series of Poker, with "**Texas hold 'em**"
- Maximum of 22 players per deck!

(Better: fewer cards in players' control, harder to cheat, faster to deal.)

1999 Poker machines in every business in Montana.

*** Choose your game! Choose your price! ***

2003 Chris Moneymaker turned $40 into $2.5 M in World Series of Poker.

2004 ESPN coverage of World Series of Poker

* Popularization through exposure, technology

2005 Here I am with a **thesis**: The game's evolution did not occur in a vacuum, social and technological developments influenced it while industrial efficiency drove adaptations of the game for maximum psychological engagement.

Technology enabling poker in America

< 1800 Riverboats exempt from state laws
< 1900 Pioneers' loneliness and isolation
1886 "Drop card" in liquor and cigar stores
1898 Charles A. Fey's 3-reel bell slot machine
- Adapted to 5-reel poker machine
- Adapted for "draw" with second pull

1920s John von Neumann and Game Theory
- **Business is just a probabilistic gamble**, too.

1950s Card games to model and test computers
* Compatible well-defined structures

1964 First electronic poker machine
≥ 1970s Video poker installed at casinos
≥ 1980s Solitaire programs on every desktop computer
- Some states legalize machine gaming in gas stations, bars, grocery stores.
- "Random Variables" enable controlled-profits on machines (assuming honest random-number generators, of course).

2004 ESPN broadcast World Series of Poker, better view of cards than players.

POKER: The rules give axioms, definitions, and theorems:

Players are dealt one card at a time, place "face down," from a randomized 52-card deck, until each player has 5 cards.

These 5 cards are called a player's "hand." A round of play is also called a "hand," so that "Each person gets one hand per hand."

Players show and compare hands to determine the winner of the round. The game places value on particularly easy kinds of combinations of cards: "Easy" because the combinations of value are both well-defined and visually identifiable, based on the equivalence classes inherent in the standard deck.

The combinations of cards valued in poker require we first know about:

Comparative-values for the 13 kinds in poker: The rules of the game assign the 13 "kinds" in the deck a (transitive) ordering:

HIGH			to				LOW
Aces,	**kings,**	**queens,**	**jacks,**	**10's,**	**... ,**	**3's,**	**2's**

So we could write, and be understood, A > k > q > j > 10 > 9 > ... > 3 > 2

This ordering is used in poker with function to:

1) determine the winner of a cut (see random decisions);
2) enable a concept/combination known as a “straight”;
3) help well-define an order on “hands of the same type”.

Types of Hands: ie. valued combinations of 5 cards

Defined for easy identification with the mind and senses, a hand is considered valuable if the five cards include:

Type 1: based on the 13 “kinds”

- Two of any one kind (“a pair” or 1P)
- Two of any two kinds (“two pair” or 2P)
- Three of any one kind (“three of a kind” or 3K)
- Three of any one kind AND two of another (called a “full house,” FH)
- Four of any one kind (“four of a kind” or 4K)

Type II: based on the four “suits”

- Five cards in any one suit (“a flush” or Fl.) (since 1840’ish)

Type Three: based on the “**ordering**” as defined on the kinds (see above)

- Five cards “in a row” (“a straight” or St.) (since 1860’ish)

Note: In poker, an ‘ace’ may be used in a ‘straight’ either as a low card in the hand {A, 2, 3, 4, 5}; or as a high card in the hand {10, J, Q, K, A}.

Type 4: none of the above (“nota”)

- Five cards not of the same suit, no two of the same kind, AND not in consecutive order

Note: A hand of renown is called a straight flush--five cards in a row AND all of the same suit (thus combining the natural and the imposed characteristics of the set, ie. pips of, and ordering on, the deck). The highest of the straight flush hands, thus the highest hand in the game of poker, includes the 10, J, Q, K, Ace in one suit, is called a “royal flush.”

Comparative value of the hands
In poker, the hands are given a (transitive) ordering:

<u>HIGH</u> <u>to</u> <u>LOW</u>
Straight Flush, 4 of a kind, full house, flush, straight, 3 k, 2 pr, 1 pr, nota

So we could write, and be understood:
SF > 4 k > FH > Fl > St > 3 of a kind > 2 pairs > 1 pair > nota

The **winner** of a round ("hand") is determined by a (physical) comparison of patterns in the cards in the hands. The player holding the "highest" hand wins.

In the case of a Tie: If two players both have a hand classified as <u>(one of the above)</u>, then that one which includes the higher kind is deemed the higher hand. **The order of the hands is nested on the order of the kinds!** (While the suits are all equal in the games poker and democracy!)

- Example: if both are flushes, then regardless of suits,
{2, 4, 6, 8, A} beats {8, 10, J, Q, K}

The Order and Probability

In poker, this ordering of hands is given in "the rules of the game" determined by Persians circa 15th century. It is not an arbitrary order!

The values of two types of hand compare inversely to the probabilities of getting them (dealt from a randomized deck), as recognized only circa 18th Century Europe. **Higher valued hands have lower probability of occurrence!**

Probabilities exist ... hmmm ... So can be computed ... so ... the game can be rationalized? (only sort of, see von Neumann) ... so commercialized ... so industrialized. Which makes it susceptible to the ultimate quests of capitalism:

*** HOW can: **I** profit from **Your** game? **I** maximize those profits? ***

BREAK TIME: Please take a moment to discuss these questions, refresh your water glass, stretch, and etc. The activity half of the workshop to follow.

Text overhead: Tom Stoppard on Chance from *Rosencrantz and Guilderstern are Dead*, a play and movie.

Back from break: How did you decide to Industrialize Live Poker?

For profit without graft ... take a "**rake**" = a percentage of each bet
- Dealers must do the math on each betting round.

To maximize the profit ...
- higher stakes? - higher rake? (percentages now regulated)
- more players? - faster games?

Change centuries of tradition?

No need to! Just use slow, user-friendly, and "seamless" upgrades: Not to change the game, just the way it is dealt! See summary of results of adaptations over the years:

From 20 cards in the game's deck to 52, by 1840:
- Same 5-card hand, same patterns
- More people could play a hand
- New possibilities: flush without straight, straight without flush--both visual, natural to deck, and embraced by players of all social ranks.
- Still viscerally comparative (corner pips came later)
- Difficulty: more "kinds" to keep track of (corner pips trivialized this concern)

From 5 cards per hand to 7, by 1860:
- Choose 5 out of the 7 cards dealt you, then play exactly like before
- People like "do overs" and enthusiastically embraced them!
- New "skills" opportunities, elevated from pure chance:
  "Betting on the come" and knowledge of probability
  "Bluffing" and knowledge of psychology
- Jump in engagement overwhelmed new abstraction: still visual, comparative.
- New difficulty: Choice and decision!
  In draw: Two cards to do over: Which ones? Is it worth the expense?
  In 7-card stud: "Is it worth the expense?" many times per hand.

Logistics of various poker games

game	max # play 20 card deck	max # play 52 card deck	max # bets	max # cards dealt for 10 players
poque	4	--	1	--
5 straight	4	10	1	50
5 stud	4	10	4	50
5 draw 2	--	7	2	7 need 49
7 stud	--	7	6	7 need 49
* Texas Hold'em		22	4	25

* Hold'em allows the most players, and need the fewest cards dealt out, and puts the fewest cards into the players' hands.

Methods of dealing and betting:

(Notice, the rules get more complex as we approach modernity...)

Straight 5-card poker:

Deal clockwise around the table, one card **face down** to each player until each has five cards. Bet, check or drop; compare hands; highest hand wins the pot.

"5-card draw" poker:

(This house's rule says draw up to 2 cards.) Deal as in straight poker. Bet, check or drop. Active players (those who "called" the highest bet) choose up to 2 "discards" from hand, give face down to dealer, who replaces them from the deck, face down. Bet, check or drop; compare hands; winner takes the pot.

"7-card stud" poker:

Deal clockwise around the table, one card **face down** to each player until each has **two** cards. Bet, check or drop.

Deal clockwise around the table, one card **face up** to each active player. Bet, check or drop. Repeat until all active players show four cards face up.

For the seventh card, deal clockwise around the table, one card **face down** to each active player. Players choose 5 of their 7 cards to call "hands."

Bet, check or drop; compare hands; winner takes the pot.

(Some house rules limit the number of betting rounds.)

“Texas Hold ‘em” poker:
- Deal clockwise around the table, one card **face down** to each player until each has **two** cards. Bet, check or drop. (Those are the only cards a player gets--all other cards will be shared “community cards.”)
- Deal three cards face up on the table, called the “**flop**.” Bet, ...
- Deal one card face up on the table, called the “**turn**” card. Bet, ...
- Deal one card face up on the table, called the “**river**” card.
- Bet, check or drop; compare hands; winner takes the pot.
    (This game includes some “forced” bets, not discussed here.)

Group Activity 1 (8 - 10 minutes, want > 5 per group):
Theoretic vs. experimental probabilities. (See page 15, theoretic probabilities.)

Generate and compile data about 5-card straight poker.
-- Each group needs to choose one person to deal cards, one person to take data, and one person to present your data at the end of this activity.
1) Play as many hands as time allows.
    - The group that documents the greatest number of hands gets a prize.
2) For each hand, the group’s recorder needs compile data in two different tables.
    - You need to document two different kinds of outcomes on each round:
        i) the type of hand dealt to each individual player, AND
        ii) the type of hand that won the round.

3) During one round, stop the dealing after everyone has been dealt exactly 4 cards. Each player should then consider two questions. Stop and discuss:
    i) What is the probability that the 5th card will be helpful to your hand?
    ii) What is the probability that you will then hold the winning hand?
Note: These represent two essentially different concerns, yet both settings are commonly described as “Conditional Probabilities.”

4) As time runs out, compile frequencies for the various hands, and be prepared to read your summary-stats aloud, so that class data can be compiled.

As “n” approaches infinity ... ?

Even while some groups’ experimental data invariably deviates significantly from the theoretic distributions, the workshop totals invariably look closer to theory.

Group Activity 2 (15 - 20 minutes, want > 5 per group):

Repeat as in Activity 1, except play Texas Hold'em.

Use a new set of tables to compile data.

In both workshops in 2005, the folks attending seemed more lively and energized during Activity 2. Both gave rise to consensus that "Hold'em" is clearly more fun and engaging than straight, five-card poker.

Thus, we have numeric evidence suggesting that Texas hold'em represents the poker game maximally-efficient for industrial to date. We generated psychological evidence that it is also the most pleasing variation for humans to date. Does that make Texas Hold'em the ultimate poker game for people and industry? *****

Handouts for workshop included the following information:

<u>The Probabilities for straight, 5-card poker hands</u>

These all use the formula:

$$\text{the probability of an event} = \frac{\text{(\# of elements in the event)}}{\text{(\# of elements in the sample space)}}$$

Let s = 2,598,960 (the number of different 5-card hands)

Type of hand	Method	Probability
Straight Flush	40/s	≈ .00001539
4 of a kind	624/s	≈ .00024010
Full house	3,744/s	≈ .00144058
Flush	5,108/s	≈ .00196540
Straight	10,200/s	≈ .00392464
3 of a kind	54,912/s	≈ .02112845
two pairs	123,552/s	≈ .04753902
one pair	1,098,240/s	≈ .42256903
none of the above	1,302,540/s	≈ .50117739

__The Number of possible ways for straight 5-card poker hands__ from a randomized standard deck of 52 playing cards.

Each computation represents its own rationale for approaching the given counting problem. There are often several different ways to discover the one absolute answer to “how many ways can you ...?”

Here, “aCb” stands for the standard computation a!/[(a-b)! b!].

Hand	Computation	Number
1 pair, no better	i) 13 * 4C2 *12C3 *4^3	**1,098,240**
	ii) (52 * 3 / 2!) (48 * 44 * 40 / 3!)	
2 pairs, no better	i) 13C2 * 4C2 * 4C2 * 11 * 4	**123,552**
	ii) [(52 * 3 /2) (48 * 3 /2) / 2!] * 44	
	iii) [(13C1 * 4C2) (12C1 * 4C2) / 2!] * 11C1 * 4C1	
3 of a kind, no better	i) 13 * 4C3 * 12C2 * 4^2	**54,912**
	ii) (52 * 3 * 2 / 3!) (48 * 44 /2!)	
	iii) 13 * 4C3 * (12 * 4C1 * 11 * 4C1 / 2!)	
Straight, no better	i) 10 * 4^5 - 40	**10,200**
	(there are 40 “straight flush” hands!)	
Flush, no better	i) 13C5 * 4 - 40	**5,108**
Full-house	i) 13 * 4C3 * 12 * 4C2	**3,744**
	ii) (52 * 3 * 2 / 3!) (48 * 3 /2!)	
4 of a kind	i) 13 * 4C4 * 12 * 4C1	**624**
	ii) (52 * 3 * 2 * 1 / 4!) * 48	
Straight Flush	i) 10 * 4	**40**
None of the above,	“nota”:	**1,302,540**

for these two methods, you need some of the above information

i) (hands without even a pair) - (hands that are flush or straight) 1,317,888 - 15,348 =

[(13 * 4) (12 * 4) (11 * 4) (10 * 4) (9 * 4) / 5!] - (10,200 + 5,108 + 40)

ii) (total number of hands) - (hands noted above as valued) 2,598,960 - 1,296,420 =

2,598,960 - (40+624+3744+5108+10,200+54,912+123,552+1,098,240)

Total number of different hands 2,598,960

Related activities, from AMATYC but omitted from San Diego presentation.

The number of different poker hands

Probabilities can be computed for five-card hands in straight poker from a randomized, 52-card deck by counting 5-card subsets of the deck and using techniques from probability theory.

Example: Probability(you get a certain type of hand, say E) = Pr(E) =

$$= \frac{\text{the number of different E hands possible in the deck}}{\text{the number of different 5-card hands in the deck}}$$

(The math involved in draw poker is a tad more involved, though leaves us with the same ordering for the hands.)

First we get the denominator … the number of "different" 5-card hands
- ie. the number of 5-element subsets of a 52-element set
- ie. the number of ways to choose 5 items from 52

By the multiplication rule for events, there are

52 * 51 * 50 * 49 * 48 = 311,875,200 ways

to be dealt five cards from the deck. But this computation implies a concern for the process that is not reflected by evaluations in the game of poker (in particular, the order in which the cards are dealt). Poker values only the finished five-card product.

Each of the "different" five-card poker hands can be dealt in

2 * 3 * 4 * 5 = 5 **!** = 120 different ways.

Activity: Let a poker hand include a 1, 2, 3, 4, and a 5. List all of the ways this hand can result, listed "first-card-dealt on left to last-card-dealt on right".

A solution:

12345	13245	14235	15234	
12354	13254	14253	15243	
12435	13425	14325	15324	
12453	13452	14352	15342	
12534	13524	14523	15423	
12543	13542	14532	15432	
2 * 3	2 * 3	2 * 3	2 * 3	= (2 * 3) * 4 = 4!

So far there are (2 * 3) * 4 = 4! ways to deal out this hand This list includes only all of the ways to get this hand dealt where the ace is the first card received by the player. We can generate four similar lists--with 2 the first card dealt, 3 first, the 4 and the 5 always first. Thus we have (2 * 3 * 4) * 5 = 5! = 120 different ways to be dealt this one particular poker hand. The same can be said for any particular poker hand.

Thus, we can find (311,875,200) / 120 = **2,598,960** different five-card hands possible from a deck of 52 cards.]

[Omitted from this presentation:

Special Cases in Comparing "Hands" of the same type:

Full-house: Compare for the highest kind in the triplet only.
Example: {A, A, A, 2, 2} beats {K, K, K, Q, Q}
Two-pair: Compare on each player's highest pair.
If that is a tie, then compare the lower pair.
If a tie still remains, then compare for highest fifth card
If a tie still remains, see "random decision"
Example: {2, 2, 3, 3, 6} beats {2, 2, 3, 3, 5}
Straight: Compare on each player's highest card.
If a tie, see "random decision"
Flush: Compare on each player's highest card.
If a tie, compare on second highest, then third, etc.
If identical kinds in different suits, see random decision
A pair: Compare for the highest pair.
If a tie, compare each player's highest other card.
If a tie, compare the second highest, then third if need.

RANDOM DECISIONS are usually best if predetermined as "House Rules." To determine a winner in a tie, players should agree to cut the deck for high (or low) card, flip a coin, draw straws, or etc. ... or "split the pot" ...]

[Omitted from this workshop, from OR-MATYC 2003.
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Activity: **Adjust a random variable to meet given constraints**.
For best results, follow along on worksheet page 22, to see what the lists show.

For this we will **use the power of the list in TI-83 calculators**. Let the calculator do what it does best--compute. We'll look at a way to minimize data entry (and the potential for typos) to work with the problem of finding appropriate random variable assignments for our machine game of one-player, 5-card, straight poker, with given constraints for our expected value.

First, we'll use the number 2,589,960 often, so want to give it an easy name. I like the letter "s." In the home screen of your calculator (use 2nd QUIT to get there) type 2 5 9 8 9 6 0 STO then ALPHA S then ENTER . (S is the letter on the LN button, in the leftmost column).

Next, we'll begin work with lists. Hit STAT then ENTER to get into the lists window.

*** We'll want to use at least 5 lists, so if not already, you should clear them. To clear all lists, use 2nd MEM ClrAllLists . To clear one at a time, arrow up until list title is highlighted, hit CLEAR then hit the down arrow. ***

In L1 enter the "number of ways" each type of hand can occur.
(40 ENTER, 624 ENTER, 3744 ENTER, 5108, 10200, 54912, 123532, 1098240, 1302540)

Right arrow into L2, then up arrow until title is highlighted.
Type 2nd L1 / ALPHA S ENTER
L2 fills with the decimal approximations for the probabilities of the types of hands.

Right arrow into L3, then up arrow until title is highlighted.
Type 1 / 2nd L2 ENTER
L3 fills with the reciprocals of the probabilities.
To satisfy the theory that "in a fair game, the payout should be inversely proportional to probability;" we can take this as a good random variable for payout. In computing the expected value of this random variable, each product-pair here is a number times its reciprocal, thus 1. With a partition of 9 elements, their sum is 9; we barely need computing support to figure that. This, however, suggests that the cost to play should be 9, which is not convenient to consumers. We can adjust our random variable's values to yield an expected value of 1, thus cost to play of 1, by simply dividing them by 9.

Right arrow into L4, then up arrow until title is highlighted.
Type 2nd L3 / 9 ENTER .
L4 fills with payout values in a fair game with cost to play of 1.

We can now use features of the calculator to find the expected value for the suggested payouts. To satisfy $E(\$_i) = \sum \$_i \cdot P(\$_i)$, we need determine the products $\$_i \cdot P(\$_i)$ and then sum them.

The traditional approach requires that we compute each of the nine products x times P(x), organize these in a table, and then add them to obtain the random variable's expected value. This method can be duplicated with the lists in the calculator. Use the lists that already contain the given information: L2 has the probabilities, L4 has the suggested "fair game" payout assignments.

One way: In LIST EDIT screen, arrow into L5, then up arrow until title is highlighted.
Type 2nd L2 * 2nd L4 ENTER . **L5 fills with the desired products x*P(x).**
To obtain the sum of these products, and thus the expected value of the game, we can:

Use the STAT CALC features of the calculator.
Hit 2nd QUIT then STAT then arrow right to CALC then ENTER = 1 = 1-Var Stats .
(This places you in home screen, where you are prompted to supply the location for a list of data.)
Type 2nd L5 ENTER to obtain basic stats on the data in L5.
This includes, on the second line from the top, $\sum x$ **,** the sum of the products stored in L5, which is **the desired "expected value"**.
or

The calculator can also give us our desired result more directly--use the sum feature. From the home screen, hit: 2nd LIST right arrow to MATH then type 5.
This places "sum(" on the home screen. This command needs a list for its argument.)
Type 2nd L5 Enter. **This sum is the desired "expected value."**

Is it the same as $\sum x$ found in the first method?

Another way: Try a shortcut: In the home screen, compute sum(L2*L4) ENTER.
(Use 2nd ENTRY and edit the argument)

This yields the same expected value as found above. It provides a method to obtain the sum of some products, without documenting those products themselves.

This time saving may become valuable when trying to achieve a random variable that will address legal concerns as well as human psychology in the programming of game machines.

Review progress, and information in your lists:
L1 has number of ways for each
L2 has probability of each
L3 has reciprocal of probabilities--fair game payout with expected value = cost = 9
L4 has one ninth L3--fair game payout with expected value = cost = 1
L5 has some intermediate info.

Trouble: The payouts for a fair game with cost 1 includes decimal fractions. This is inconvenient for the machine, it wants only multiples of the cost to play.

Activity: In search of expected value of 1 with whole number payout values.

1) In L5, round to whole number values to approximate the values in L4.

2) Check the expected value by the following key strokes:
2nd QUIT 2nd LIST arrow right to MATH then hit 5 = Sum(
2nd L2 * 2nd L5) ENTER

3) Adjust the list in L5 (hit STAT ENTER, arrow into L5) to bring us closer to expected value of 1, then check via:
2nd QUIT 2nd ENTRY ENTER

4) Repeat steps 2 and 3 until you've got a sum of products equal to 1.

Activity: In search of random variable assignments for an expected value of 0.86.

1) In L6, enter values (use the fair game values in L5 as guidance) to guess.

2) Check the expected value by the following key strokes:
2nd QUIT 2nd ENTRY and edit Sum argument to "L2*L6" then hit ENTER.

3) Adjust list to get closer (hit STAT ENTER, arrow into L6).
Check via 2nd QUIT 2nd ENTER ENTER.

4) Repeat step 3 until sum is very close to 0.86 and payouts seem reasonable.

Partition, Probabilities, potential Fair Game random variables.

Recall: A fair game is one in which the "expected value" is equal to the "cost to play." In a game governed by random variable "$\$_i$" the expected value is $E(\$_i) = \sum \$_i \cdot P(\$_i)$

TYPE	PROBABILITY	1/P($)	(1/P($))(1/9)	Rounded
		rv	rv	rv
Straight Flush	40/s ≈ .000015	64974	7219.3	1500
Four of a kind	624/s ≈ .000240	4165	462.8	400
Full House	3744/s ≈ .00144	694	77.1	75
Flush	5108/s ≈ .00197	509	56.5	50
Straight	10200/s ≈ .00392	255	28.3	25
Three of a kind	54912/s ≈ .0211	48	5.3	5
Two Pair	123532/s ≈ .0475	21	2.3	1
One Pair	1098240/s ≈ .420	2.37	0.26	1
N.o.t.A.	1302540/s ≈ .501	2	0.22	0
	Expected Value:	9	1	0.999
	Fair Game cost to play	9	1	1

ACTIVITY: One state requires gaming machines to give a minimum of 86% return to players. This translates into a game with a cost of 1 to have expected value of 0.86 at least. Devise a random variable which yields this maximum profit (14% as house take) in the long run (assume reasonably good random number generators).

TYPE	Prob.	guess			
S.F.	≈ .000015				
4 K	≈ .000240				
FH	≈ .00144				
F	≈ .00197				
S	≈ .00392				
3 K	≈ .0211				
2 Pr	≈ .0475				
1 Pr	≈ .420				
N.o.t.A.	≈ .501				
Expected Value:					
	adjust?				