Proofs for 1 = .999…
in Developmental Mathematics

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Motivations and Objectives

Very often, developmental students are bogged down with rudimentary ideas of mathematics, creating an illusion that math is about steps, procedures, computation, and following directions. This exercise helps students to review some developmental computation, basic algebraic operations, and ultimately lead students to create the process of converting repeating decimals to fractions. By engaging an exercise like this, the students would have done math in a much different manner than those of a typical developmental math activity.

Arithmetic Proof 1

This activity is designed to strengthen the understanding of the fraction-decimal connection by reviewing fraction-decimal conversion and fraction & decimal addition.

Objective: Given $\frac{1}{9} = 0.\overline{1}$, then students gradually discover the decimal equivalent of $\frac{9}{9}$.

$$\frac{1}{9} + \frac{1}{9} = 0.\overline{1} + 0.\overline{1} = 0.\overline{2}$$

$$\underbrace{\frac{1}{9} + \cdots + \frac{1}{9}}_{9 \text{ times}} = \underbrace{0.\overline{1} + \cdots + 0.\overline{1}}_{9 \text{ times}} \text{ OR } \frac{9}{9} = 0.\overline{9}$$

Hence, $1 = 0.\overline{9}$.

Other useful fractions for this activity includes:

- $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=0.\overline{3}+0.\overline{3}+0.\overline{3}$
- $\frac{1}{11}+\frac{10}{11}=0.\overline{09}+0.\overline{90}$
  Or $\frac{m}{11}+\frac{n}{11}=\frac{11}{11}$ where m and n are whole numbers.
- $\frac{1}{7}+\frac{6}{7}=0.\overline{142857}+0.\overline{857142}$
  Or $\frac{m}{7}+\frac{n}{7}=\frac{7}{7}$ where m and n are whole numbers.
- $\frac{1}{6}+\frac{5}{6}=0.1\overline{6}+0.8\overline{3}$

Arithmetic Proof 2

This activity is designed to strengthen the understanding of the fraction-decimal connection by reviewing fraction-decimal conversion and fraction & decimal multiplication.

Objective: Given $\frac{1}{9} = 0.\overline{1}$, then students discover the decimal equivalent of $\frac{9}{9}$ by multiplying a reciprocal.

$$\frac{1}{9} \times 9 = 0.\overline{1} \times 9$$

$$\frac{9}{9} = 1 = 0.\overline{9}$$

Again, other one may use other fractions with repeat decimal equivalent for this activity.

Algebraic Proof

This activity is designed to strengthen the understanding of the equation operations. Simultaneously, the proof requires the prerequisites of multiplication by power of 10 and subtraction of repeating decimals.

Objective: Given $x = \frac{1}{9} = 0.\overline{1}$, then students discover the equation manipulation would lead to $1 = 0.\overline{9}$.

$$x = 0.\overline{1}$$

$$10x = 1.\overline{1}$$

$$10x - x = 1.\overline{1} - 0.\overline{1}$$

$$9x = 1$$

$$x = \frac{1}{9}$$

The Discovery of Repeating Decimal to Fraction Conversion

At this point, it is important to make the students aware that what not only they have shown that $1 = 0.\overline{9}$, they also have established that if $x = 0.\overline{1}$, then $x = \frac{1}{9}$.

Although this may seem to be a side point to the students, it is important to point out to the students that this is a significant discovery of converting a repeating decimal to a fraction, and one may discuss the possibility of brainstorming a technique based on the proof to devise a procedure for the conversion.

At the end of the discovery process, students may be debriefed by reviewing what techniques are necessary in order to execute the conversion successfully:

- Multiplication by the power of 10
- Vertical subtraction with repeating decimals with a resulting difference as an integer

Some Typical Repeating Decimals For Students to Convert

- $0.\overline{4}$
- $0.\overline{63}$
- $0.\overline{09}$
- $0.2\overline{5}$
- $2.\overline{1}$
- $7.\overline{296}$
- $3.\overline{001}$
- $9.01\overline{8}$

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