

## Enriching Calculus with Formative Assessment Activities

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Electronic copies of activities are available at the Wake Tech website, Math & Physics Department, faculty presentations. <http://mathandphysics.waketech.edu/displayfiles.php>

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How do students assess their knowledge? How can students plan for closing learning gaps? Formative Assessment provides layers of organization and pre-planning guidance to reach instructional goals for student learning outcomes.

The goals for this session include having participants identify key elements of Formative Assessment, discuss examples of learning activities using elements of Formative Assessment, and use a peer observation rubric to assess their own use of Formative Assessment in the classroom.

Definition Formative Assessment: “Formative assessment is a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students’ achievement of intended instructional outcomes.” ATTRIBUTES OF EFFECTIVE FORMATIVE ASSESSMENT, McManus, NC Department of Public Instruction

Good teachers are always assessing the classroom learning experience and making adjustments. In contrast, students often feel they are doing all right in a class, and then they have a graded assessment and find out otherwise. Teachers need to actively plan for ways to engage students in assessing their individual progress and making their own on-going adjustments to achieve learning goals.

Formative Assessment for students reminds faculty to put information in the students’ hands. Students should be able to answer the questions: what do I know, what do I need to know, how can I close the gap?

One tool to aid in lesson preparation is a Formative Assessment Rubric or Attribute Chart. This chart is a planning and observation instrument developed, tested, and revised by the Wake Tech Mathematics and Physics Department. The chart identifies four key elements with strategies for formative assessment:

- Involving students in their learning
- Modeling quality
- Giving feedback
- Student self-assessment opportunities

## Formative Assessment Planning & Peer Observation Attribute Chart Wake Tech CC

Please make a check in “Observed” column for each strategy observed. Space is provided for notes for later discussion between the peer partners.

<b>Involving Students in Their Learning</b>	<b>Observed</b>	<b>Notes</b>
1. Goal for lesson stated and motivated		
2. Students’ contributions encouraged and built upon		
3. Individual tasks designed to enable students to “move along” by themselves		
4. Group activities planned that enable students to collaborate on a joint outcome		
5. Specific comments given to students such as they are on the right path and in what way		
<b>Modeling Quality</b>	<b>Observed</b>	<b>Notes</b>
1. Students asked to demonstrate to others how they did something		
2. Students encouraged to suggest ways a solution can be improved		
3. Formats and structures provided for writing and recording findings		
4. Students were shown a range of other students’ work to assist them in evaluating their own work.		
<b>Giving Feedback</b>	<b>Observed</b>	<b>Notes</b>
1. Probing questions used to diagnose the extent of the students’ learning		
2. Completed work analyzed to work out why a student has or has not achieved		
3. Support given when achievement is satisfactory		
4. Students given constructive corrective feedback.		
5. Students were told what they have and have not achieved with specific reference to their learning		
<b>Student Self Assessment Opportunities</b>	<b>Observed</b>	<b>Notes</b>
1. Students asked to review their own work, record their progress, and suggest ways they can improve		
2. Students guided to improve a process		
3. Time provided for students to reflect and talk about their learning		
4. Students helped to understand their achievements and know what they need to do next to make progress		

## Formative Assessment Examples

Documents available by e-mail send request to [mdpearce@waketech.edu](mailto:mdpearce@waketech.edu); [sfwelker@waketech.edu](mailto:sfwelker@waketech.edu)

### **Formative Assessment Resources**

Planning & Observation Attribute Chart

Bloom's Polygon: verbs & activities to reach variety of learning levels

Sun Diagram: visualization of key elements

Resource list and definitions

### **Class Activities**

Functions of Morris & Mavis: First Day Interactive Matching Activity

Blackboard Announcements and External Links: take time to open websites, provide snapshots

Quiz Ideas: warm-up, limit to 2 problems, group work could extend, reflection quizzes

Understanding Derivatives: Excel slider, video writing assignment

Related Rates: catch their attention, connect animations to lecture & group work

Optimization: format and structure provided for a process

Fundamental Theorem of Calculus: exit internet activity, Critical Thinking, follow-up assessment

### **Projects**

Project prep: individual commitment to contribution, review or warm-up

Project Contract: groups determine expectations and criteria for membership

Project: clear goals, project outline, assessment rubric

Project check-in: key portion of project is due one week before final due date, group feedback

Project follow-up: minute paper including mathematical insight and feedback

### **Tests**

Top 10 List: students identify learning objectives prior to review day

Minute paper: information for instructor, information for student

Mind Map: individual, small group, or whole class

Test reflection: reflect on where information was available, how can it be used more effectively

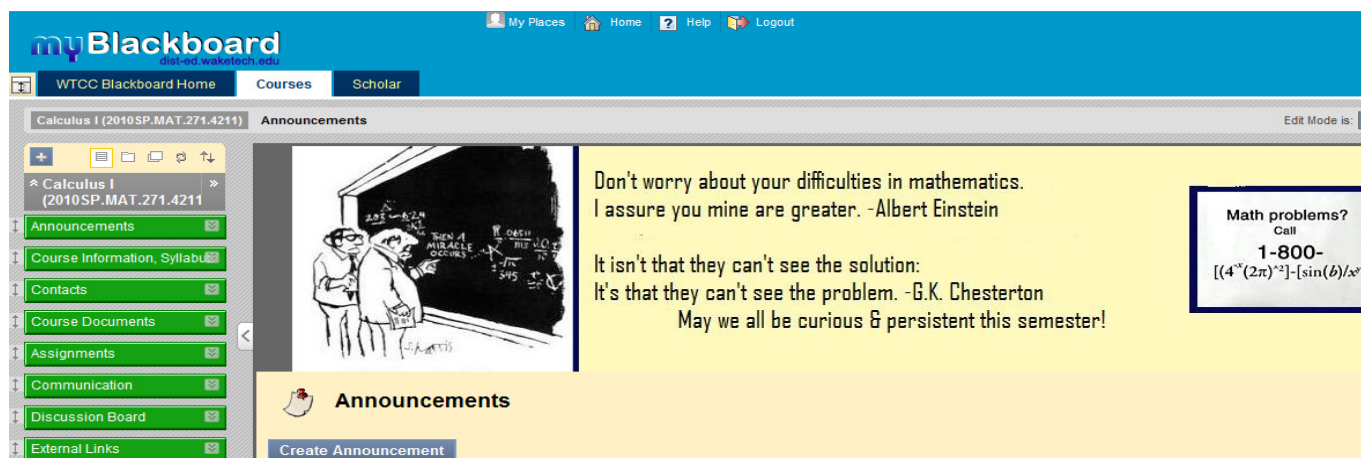
## Classroom-Ready Ideas

### AMATYC Conference 2010: Enriching Calculus with Formative Assessment Activities

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#### Using Blackboard as a Formative Assessment Tool

Type a quick announcement after each class to summarize key objectives, or class activities, reminding students what occurred in class and how they can build on that work to learn the concepts and successfully complete assignments.



The screenshot shows the Blackboard interface for a course titled "Calculus I (2010SP.MAT.271.4211)". The announcement is titled "Announcements" and features a cartoon illustration of two men in a classroom setting. The text of the announcement reads: "Don't worry about your difficulties in mathematics. I assure you mine are greater. -Albert Einstein" and "It isn't that they can't see the solution: It's that they can't see the problem. -G.K. Chesterton". Below this, it says "May we all be curious & persistent this semester!". On the right side of the announcement, there is a box with the text "Math problems? Call 1-800-[(4\*(2π)^2]-[sin(b)/x^2]". The interface also shows a navigation menu on the left with options like "Announcements", "Course Information, Syllabus", "Contacts", "Course Documents", "Assignments", "Communication", "Discussion Board", and "External Links".

#### Example 1

##### Precalculus Review (week 1)

You will be assessed on your pre-requisite learning with a test on the 2nd Friday of the course, Jan. 15.

##### Tools to help you focus your review:

- (1) **Text**, Chapter 1 Review, pages 73-75 and text videos
- (2) Precalculus Review **Handout** (posted in this folder, with answers)
- (3) **WA (Web Assign)** Practice problems and a graded assignment
- (4) *External links*, explore and find sites that fit your learning preferences. Some good sites to start with: [www.mathtv.com](http://www.mathtv.com), you can search *You Tube* (but be careful, not all sites give the correct procedures) see *Khan Academy*, *algebasics.com*, and *hippocampus.org*

#### Example 2

**Related Rates:** What a fun class today with ships pulling away from each other, oil spilling, grain silos filling up! Related rates problems are a very specific type of application, visualization of related rates will be very helpful. Please take advantage of the powerful animations available to you, I've posted some links with Lab 4. You do have to teach yourself the common settings for this type of problem. By the time you watch videos/animations, read the text examples, try text HW and WA with the practice helps, you should be ready for working on Lab 4.

While Announcements are good, a Formative Assessment improvement is to build on the announcements or in-class statement about website tools available by taking a few minutes to demonstrate going to the suggested website. Walk students through the search for a needed topic such as Derivatives, together observe the results.

## Quiz Ideas

### Quiz on the Quotient Rule

First Person's Name \_\_\_\_\_

- A. Create a Function that requires using the Quotient Rule to differentiate.

$$f(x) =$$

Second Person's Name \_\_\_\_\_

- B. Evaluate the derivative of the function created above.

$$f'(x) =$$

Third Person's Name \_\_\_\_\_

- C. Analyze using complete sentences the function created in part A and grade the answer in part B.

### Related Rate Scenario Warm-up

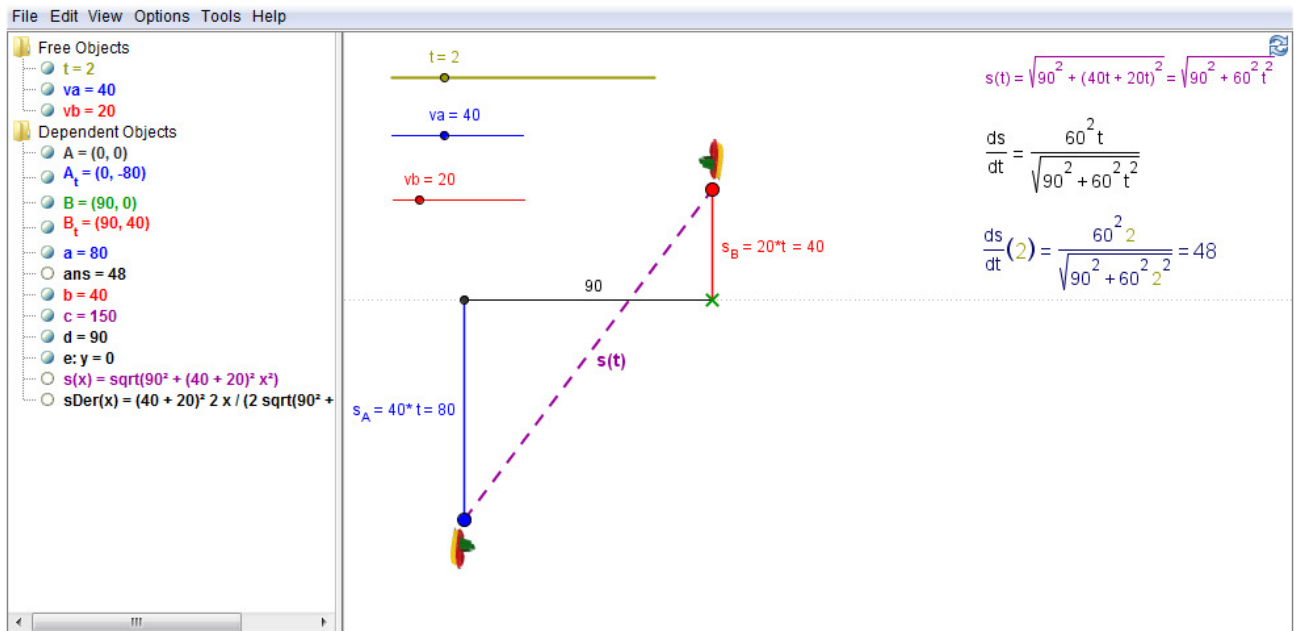
At noon, ship A is 90 miles west of ship B. Ship A is sailing south and ship B is sailing north.

- a. Draw a picture of the ship positions at 4 pm, including a line between ship A and ship B.
  
  
  
  
  
  
  
  
  
  
- b. Let **A** be the distance in miles traveled by ship A and **B** be the distance in miles traveled by ship B. Write a mathematical equation for the distance **d** in miles between Ship A and Ship B at 4 pm.

## Related Rate Scenario

<http://www.geogebra.org/cms/> and search for Calculus Related Rates

<http://mathcasts.org/gg/student/calculus/RelatedRates/rates.html>



[www.hippocampus.org](http://www.hippocampus.org), go to Calculus, Calculus AB, Applications, Defining the Problem. An interactive site that allows you to hear as well as see, you can pause and go back, or click forward to move more quickly through the material

HippoCampus.org

4. APPLICATION OF THE DERIVATIVE  
Lesson 27: Related Rates

AB CALCULUS

Defining the problem

In a baseball game, a ball is hit down the first baseline, as shown here.

As soon as the batter hits the ball, a base runner on third base starts running towards home plate.

The first baseman fields the ball and throws it towards home plate.

As the base runner and ball move towards home plate, the distance between them changes.

If we wish to determine the rate at which the distance...

## The Definition of the Derivative

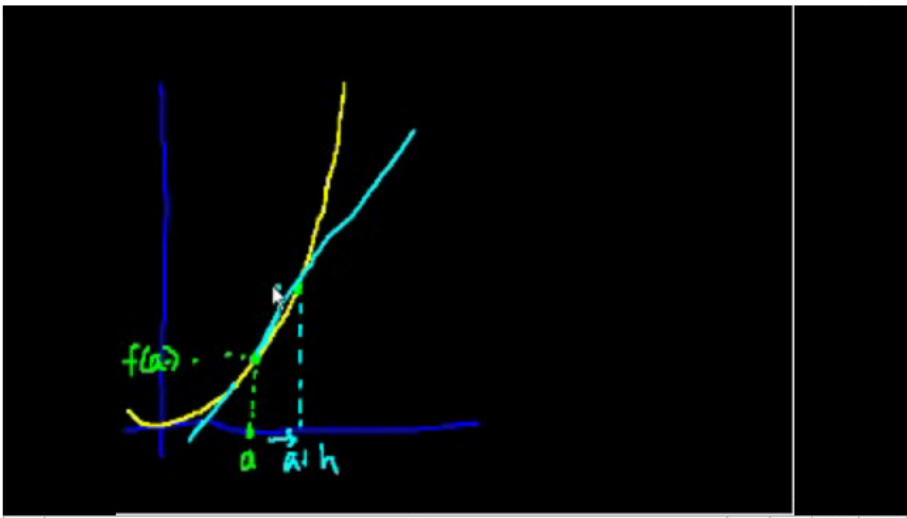
Tools to put learning in the students' hands & minds

(1) Writing Assignment: After viewing two videos as noted, write a 1-2 page paper examining the context and summarizing some applications of the two branches of Calculus. Complete a follow-up assessment. Go the following web site <http://www.learner.org/resources/series42.html>. You can scroll down to see the videocassette icons, watch #3 Derivatives and then #7 Integration. The first video, #3 Derivatives, is a 20-min video. The last part of the video gets into rules of derivatives which we will cover in chapter 3, the first part helps explain the "why" of the work we will doing in chapter 2. I want students to notice the language (derivatives), the applications (what context or examples are used to show the real-world use of calculus), and to see some of the symbols you will be learning about over the next weeks. Go back to the website, and choose video #7 Integration. This is longer, about 27 minutes, and it gives an interesting & realistic biography of Newton & Leibnitz, and an overview of the ideas of integration (our chapter 5). Again, do not try to write any formulas they present the last 10 minutes, but keep watching for the historical insight and the key application of integration. The paper should have a brief introduction with credit to the source, and a brief conclusion. One paragraph is to discuss the idea behind Differential Calculus, giving two to three examples from the video that help explain this. Another paragraph is to discuss the idea behind Integral Calculus, also with two to three examples from the video.

(2) Visualization

Home Page Series 42

### Calculus: Derivatives 1



6:03 / 9:25

★★★★★ 464 ratings

142,624 views



## Consumer and Producer Surplus

### PART ONE

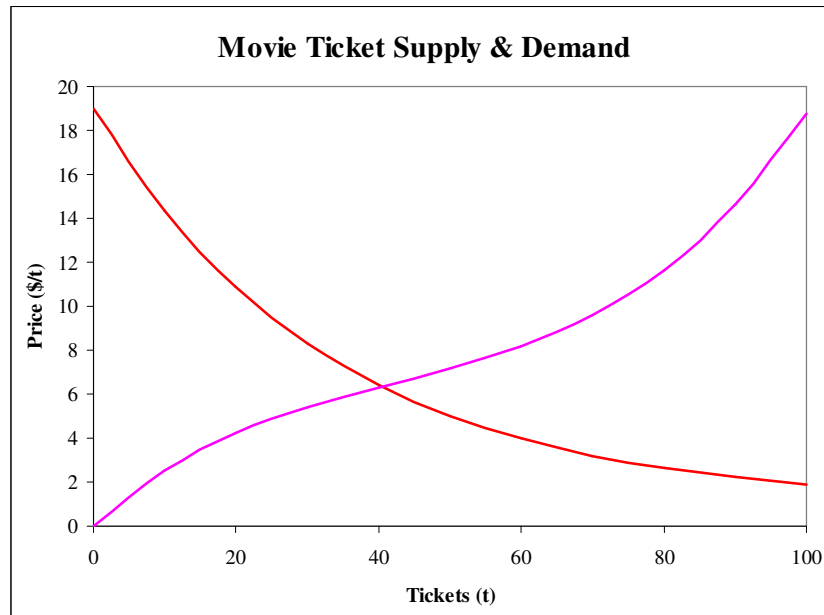
The management of the Starlight Stadium movie theater in Anderson, SC has determined that the price demanded,  $D$ , is related to the number of tickets,  $t$ , by the function

$$D(t) = 18e^{-0.03t} + 1 \text{ dollars.}$$

Starlight Stadium also knows that it wants to supply  $t$  tickets if the unit price is

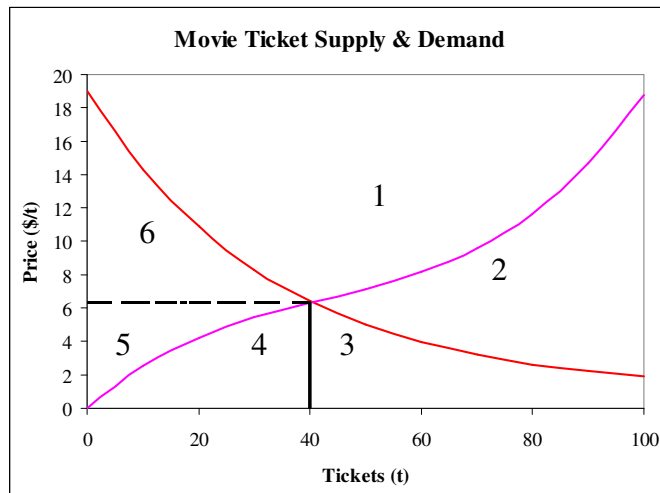
$$S(t) = 0.00004t^3 - 0.0051t^2 + 0.2979t \text{ dollars.}$$

- Find the equilibrium point for Starlight Stadium movie theater.
- Find the revenue at that point.
- The figure below shows the supply and demand curves for Starlight Stadium. Draw in the rectangle whose area represents the revenue at the equilibrium point.



## PART TWO

As you know, some consumers are willing to spend more for the item than the equilibrium price. Hence they benefit from the lower price.



The figure above shows the supply and demand curves and the equilibrium point for Starlight Stadium movie theater. Notice that there are six bounded regions. When charged the equilibrium price, the money that consumers, as a whole, will save when purchasing products at a price lower than they would demand is called the **consumer surplus**.

- Which region above represents the consumer surplus? \_\_\_\_\_
- Find the area of that region. In other words, find the consumer surplus. Describe your steps and show all work below.

## PART THREE

On the other hand, some producers are willing to sell the items for less money than the equilibrium price. Thus they benefit from the higher price.

When charging the equilibrium price, the extra amount of money that producers, as a whole, make when selling a product at a price higher than what they would supply is called the **producer surplus**.

- Which region above represents the producer surplus? \_\_\_\_\_
- Find the area of that region. In other words, find the producer surplus. Describe your steps and show all work below.

## **MAT 271 Springs Project Preparation**

**Objective 1:** Students will conduct an experiment that will produce data for the relationship that exists between time and the position of the weight attached to the spring.

**Objective 2:** Students will model the data, and analyze with calculus techniques various aspects of the data.

**Objective 3:** Students will be contributing members of a team to discuss and produce a written paper demonstrating the ability to communicate correct mathematical concepts.

As you prepare to participate in an extended group project, think about the contributions you can make, and the expectations you have for the other group members.

**Name** \_\_\_\_\_

Contributions I can make to the group (circle all that apply):

Calculus Content/Concepts

Precalculus background

Excel skills

Writing skills

Typing/formatting skills

Organizational skills

Communication skills

Other: \_\_\_\_\_

Other class members I would like to work with:

### **Group Project Fall 2010**

Draft a team contract to include:

- \* Full names of team members
- \* Goals and objectives of project
- \* Timelines for project with milestones
- \* Number of times team will meet (include outside of class):
- \* Performance expectations, i.e. attendance at meetings, timeliness for on-line responses, submission of work assigned, etc.
- \* Terms by which a team member can be terminated.

SIGNATURES of group members:

**MATH 271 PROJECT 1 GRADE SHEET**

**NAMES:** \_\_\_\_\_

	Total points possible	Points earned
First Day initial response	7	
Introduction Paragraph of Project Report	5	
Detailed explanation of how you gathered data	5	
Partial Table of Distance and Time data	5	
Scatter plot of collected data	5	
Detailed explanation of how you modeled your data	10	
Graph of collected data with model	5	
Velocity and Acceleration functions given symbolically and graphically	10	
Maximum value of velocity and acceleration and how you found them and prediction for 5e	7	
Sign chart for velocity and acceleration	7	
Conclusions about the motion using sign charts	7	
Discussion comparing your initial observations and your results	5	
Additional experiment design and hypothesis	7	
Conclusion - discussion of what you learned	5	
Overall Presentation, Coherency, Grammar	5	
Individual accountability/intermediate check-in assignment	5	
Total Points	100	

Notes: This is not a “yes/no” checklist, but a guide as to how points will be awarded for quality work.

<i>(Alternate Style of Grading Rubric)</i> Criteria						Points	
	Exemplary	Strong	Adequate	Needs Improvement	Inaccurate Or Omitted	Points Possible	Points Earned
<b>Format</b>	The whole paper is presented in paragraph format, is well written, interesting and informative to the reader.	Is an acceptable paper with no major flaws but could be noticeably improved.	Contains a cover sheet, introduction and conclusion.	Not typed, graphs and symbolic work not included in the body, missing the introduction or conclusion.	Major flaws or more than one omission.	15	_____
<b>Differential Equation</b>	Correct equation presented well with a supporting explanation	Correct equation with an insufficient explanation.	Correct equation with an incorrect explanation.	Correct equation with no explanation.	Incorrect equation	20	_____
<b>Symbolic Work</b>	No math errors, clear and concise presentation that can be easily followed.	No mathematical errors, but the work is not easy to follow or clearly explained.	No errors but no explanation or small mathematical errors with a strong explanation	Small mathematical errors with no explanation or more numerous errors in solution.	Major math errors, serious flaws in reasoning, completely missing.	20	_____
<b>Iterative Formula &amp; graphical Solution</b>	Formula and graphical solution are present and correct with detailed correct supporting explanation	Formula and graphical solution are present and correct with insufficient explanation	Formula and graphical solution are present and correct with little to no explanation	Formula and graphical solution are present but have small errors	Omitted or have major errors impacting their usability	10	_____
<b>Answer to all questions for first situation</b>	Correct answers to all questions with supporting explanations	Correct answers to all questions with insufficient explanations	Correct answers to all questions with incorrect explanations	Correct answers to all questions with no explanation or incorrect answers to some questions with a supporting explanation	Major Errors or completely absent	10	_____
<b>Answer to all questions for second situation</b>	Correct answers to all questions with supporting work and explanations	Correct answers to all questions with insufficient supporting work and explanations	Correct answers to all questions with incorrect work or explanations	Correct answers to all questions with no explanation or incorrect answers to some questions with a supporting explanation	Major Errors or completely absent	25	_____
<b>Total----&gt;</b>						100	_____

I. Consider item #4 from the project description:

4. *Graph the data and find a symbolical model for the position data. Confirm the model fits the data by graphing the data and the model together. (Hint: Model the maximum points for each oscillation with an exponential and model the oscillation with a sine or cosine function. The exponential function is the variable amplitude for the sine or cosine function.)*

**Assignment:** Provide two (2) paper copies of the GROUP work on this item #4, including the model (the equation). You do not have to provide the written description, or the explanation paragraphs, at this time.

- Partial Table of Distance & Time data
- Scatter plot of collected data
- Model
- Graph of collected data with model

II. **Individually** complete the following:

1. Identify the mathematical purpose of the project:

2. So far our group has worked together \_\_\_\_\_ times, I attended \_\_\_\_\_ times.

Our group uses emails and other distance contacts effectively: Yes or No? \_\_\_\_\_

List your name, and the names of the others in the group. On a scale of 1 to 5, very low participation to highest possible participation, rate yourself and others as to group interaction and collaboration by circling the appropriate number.

First Name	Very low	Low	Moderate	High	Very High
	1	2	3	4	5
	1	2	3	4	5
	1	2	3	4	5
	1	2	3	4	5
	1	2	3	4	5

3. A rating scale cannot give all of the information that you may wish to convey regarding your group interactions. If you would like, you may add additional comments and identify group process strengths or problems, or indicate situations that need to be addressed.



## Test Prep Activity

Provide 4-6 problems from past tests. Students prepare the solutions prior to review day, share solutions on review day

## Test Prep on or before Review Day

Are you left-brain or right-brain? Choose one of the options below, complete the task, pair with others so the group has a mix of Lists and Mind Maps.

--Write a Top 10 List of learning objectives for this test. "Students will be able to...."

--Create a Mind Map for concepts and problems for this test

**Test Prep Minute Paper:** at the beginning of the review day students complete the prompt below.

*Consider the topics that will be on the test (see Table of Contents).*

*I confident about....., or I will do well with questions about.....*

*I am not sure about.....*

*What is my muddiest point, the concept I most need to review?*

(As I collect these, I point out that students now know how to focus their study time, reinforcing what they know well, spending more time to learn their muddiest concepts. They may need to view videos, use the Web Assign practice options, work in a study group, etc..In class, with our limited review time, I'll balance topics from students noted on the minute papers)

## Top Ten Learning Objectives

In the table below list the top ten learning objectives that have been discussed in class for this test. Rate your understanding of each objective on a scale from 1 (need to spend more time on ) to 5 ( understand completely).

Learning Objective	Confidence Rating

## Functions by Words

1. Morris wanted Road Runner access and found an ad enticing him with \$25.95 a month for the first six months. His wife, Mavis, read the whole ad and found out the price increased to \$39.95 a month after the first six months. They decided to try it for a year.
2. The amount of Mavis's speeding fine depends on how fast she is driving. Her fine in dollars increases \$15 for every mile per hour she is going over the speed limit.
3. Morris's New Year's resolution was to keep track of the daily receipts at his business, the local movie theater, and make employee scheduling decisions based on the results. He found the daily receipts in dollars fluctuated with the highest totals on the weekend and the lowest during the weekdays.
4. Morris and Mavis are getting a \$2250 tax refund from the federal government. They decided to deposit it in a savings account to save for a "rainy" day. Online they found a bank that was advertising 4.65 % APR compounded monthly with no fees.
5. Mavis loves to watch nature programs. She was sorry to hear about how the habitat of polar bears is decreasing each year causing the population of polar bears to decrease. She read that the current population is approximately 21,500 and that by 2050 they could be extinct.
6. Morris noticed that the movie blockbuster Star Trek started out great, but that the weekly receipts declined at a rate of 40% during the seven weeks that it was in his movie theater.
7. Morris and Mavis moved into Antioch County in 1940 and at the time the population was increasing slowly and leveled out in the 1960s. In 1972 a new manufacturing company moved into town and since then the population has increased at a fast pace.
8. Morris's auto mechanic told him that there is a relationship between tire pressure in pounds per inches squared ( $\text{lb}/\text{in}^2$ ) and the life of a tire in thousands of miles. If they are under inflated or over inflated the life is reduced. To keep the life over 50 thousand miles the mechanic suggested that Morris keep the tires inflated to between  $25 \text{ lb}/\text{in}^2$  and  $45 \text{ lb}/\text{in}^2$ .

## Functions by Tables

a.

input	output
0	21.5
5	19
10	16.5
15	14
20	11.5
25	9
30	6.5
35	4
40	1.5

b.

input	output
0	14530
10	21460
20	22545
30	22946
40	30328
50	49651
60	86447
70	147359
80	238552

c.

input	output
55	0
60	75
65	150
70	225
75	300
80	375
85	450
90	525
95	600

d.

input	output
20	9
24	39
28	60
30	67
35	75
38	72
40	68
45	48
50	15

e.

input	output
0	2250
5	2693
10	3223
15	3858
20	4617
25	5527
30	6615
35	7917
40	9476

f.

input	output
0	690
3	960
7	2425
9	2475
12	1345
14	690
16	689
20	2160
21	2500

g.

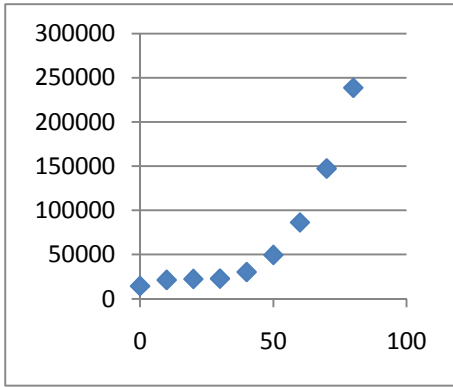
input	output
0	7525
1	4512
2	2746
3	1625
4	974
5	583
6	349
7	215
8	125

h.

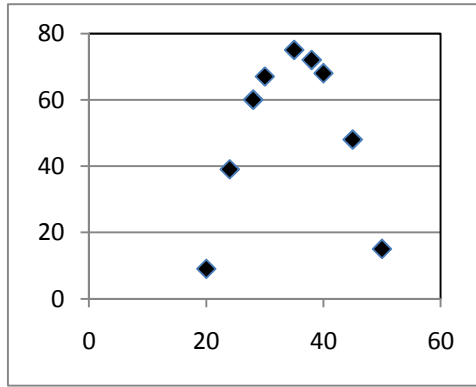
input	output
0	0.00
2	51.90
4	103.80
5	129.75
6	155.70
7	203.65
8	245.60
10	327.50
12	395.40

## Functions by Graphs

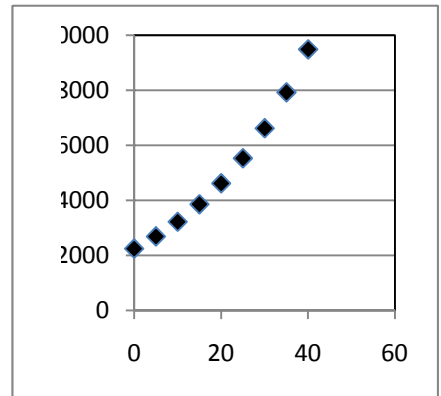
I.



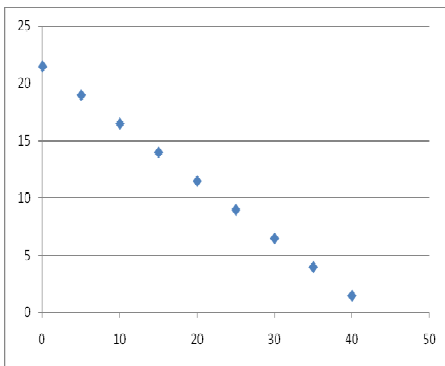
II.



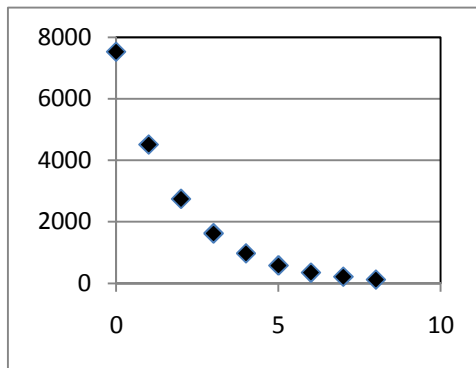
III.



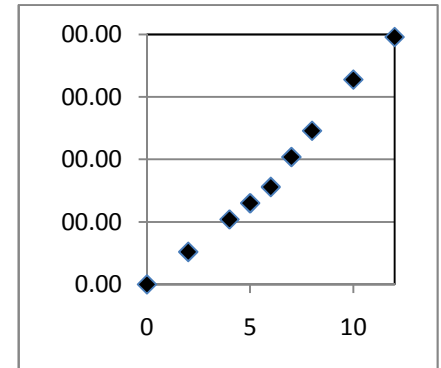
IV.



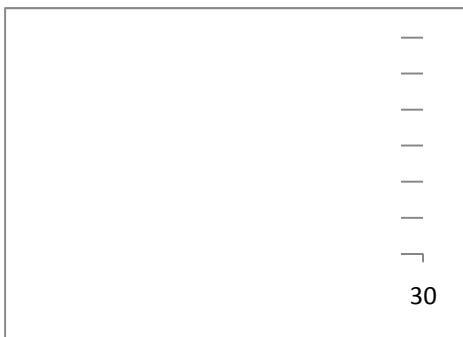
V.



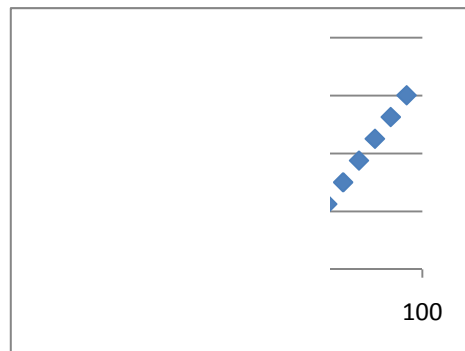
VI.



VII.



VIII.



## Functions by Formulas

A.  $F(s) = 15s - 825$

B.  $L(p) = -0.28p^2 + 19.75p - 273.55$

C.  $P(x) = (x - 20)^3 + 22545$

D.  $B(t) = 2250(1.003)^{12t}$

E.  $P(t) = -0.5t + 21.5$

F.  $T(m) = \begin{cases} 25.95m & \text{for } 0 \leq m \leq 6 \\ 39.95(m - 6) + 155.70 & \text{for } 6 < m \leq 12 \end{cases}$

G.  $R(w) = 7500(.6)^w$

H.  $D(t) = 962.35 \sin(51.43(t - 4.5)) + 1558.55$

Functions of Morris and Mavis

Group Names \_\_\_\_\_

Work as a group to match the appropriate table, graph, and formula to the word problems. Use complete sentences to justify your choices.

<b>Words</b>	<b>Tables</b>	<b>Graphs</b>	<b>Formulas</b>	<b>Justification</b>
<b>1</b>				
<b>2</b>				
<b>3</b>				
<b>4</b>				
<b>5</b>				
<b>6</b>				
<b>7</b>				
<b>8</b>				