Research on Students' Reasoning and Sense Making: Some Examples that Provide Connections Between Research and Practice

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Why might it be good idea to research student thinking in mathematics courses in Two Year Colleges?

Student Thinking and Reasoning can

- Inform us about where students are in their development of mathematical concepts
- Provide roadmaps, or at least starting points, for hypotheszing learning trajectories
- Help us plan instruction so as to build on what students *do know*
- Provide a valuable resource tool for thinking about, curriculum development

Goals for this talk

- Share 3 examples which help us learn from documenting student thinking (one each from algebra, geometry, and statistics)
- Put you in the position of wrestling with some tasks yourself— with a chance to talk to colleagues seated around you
- Point out some of the benefits of this type of research—what we can learn from listening to our students

Research Scenario 1: Algebraic Representations

Beliefs about Algebraic Reasoning

How do teachers rate student success on ‘start unkown’ and ‘result unknown’ tasks?

Work by Mitchell Nathan and

Kenneth Koedinger. JRME, Vol 31, #2

Two versions of a ‘start unknown’

1. When Ted got home from his waiter job, he multiplied his hourly wage by the 6 hours he worked that day. Then he added the $66 he made in tips and found he earned $81.90. How much per hour does Ted make?

   (story)

2. Solve for X: $X*6 + 66 = 81.90$

   (symbolic equation)

Two versions of a ‘result unknown’

3. Solve for X: $(81.90 - 66)/6 = X$.

(symbolic equation)

4. Starting with 81.90, if I subtract 66 and then divide by 6, I get a number. What is it?

(word equation)

Teacher beliefs—easiest to hardest

Teacher Beliefs

1. RU Symbolic equation
2. RU Word equation
3. RU Story
4. SU Symbolic equation
5. SU Story
6. SU Word equation

Teacher beliefs vs student performance—easiest to hardest

Teacher Beliefs	Student Performance
1. RU Symbolic equation	RU Story
2. RU Word equation	RU Word equation
3. RU Story	SU Story
4. SU Symb. equation	SU Word equation
5. SU Story	RU Symb. equation
6. SU Word equation	SU Symb. equation

Student performance in percentage

RESULT UNKNOWN	STORY 73 %	WORD EQUATION 67 %	SYMBOLIC EQUATION 53 %
START UNKNOWN	59 %	54%	37 %

Researchers main finding

“ The most salient discrepancy is teachers’ predictions that story problems and word equation problems would be more difficult than symbol-equation problems, whereas students found symbolically presented problems most difficult”

--Nathan and Koedinger JRME, Vol 31, #2

Research Scenario 2: Characterizing Reasoning about geometric shapes

How do students use clues about a shape to determine what the shape must be?

Work by William Burger & Michael Shaughnessy

JRME Vol 17, #1

What's My Shape?

I will reveal clues one at a time about a shape.

When you know for *sure* what the shape is, write down the number of that clue.

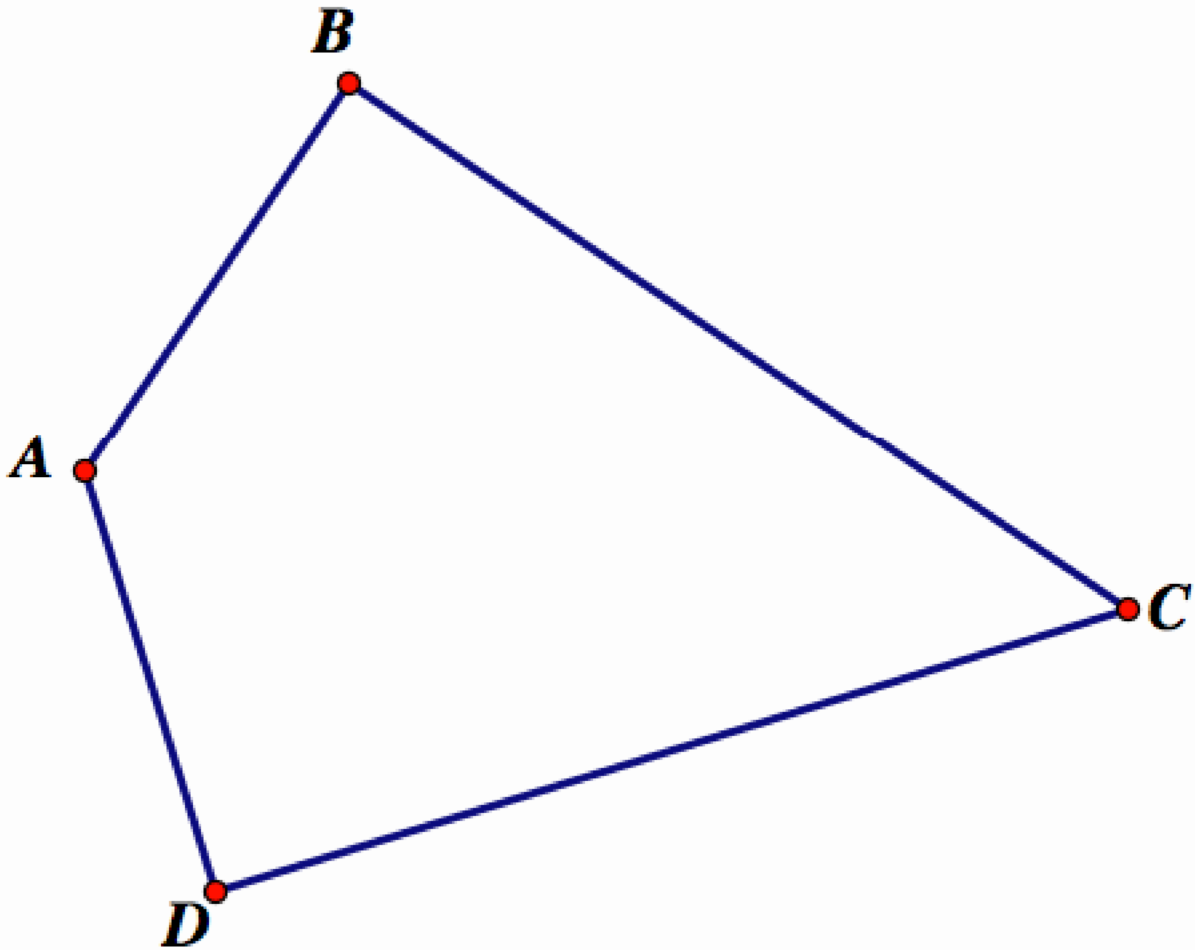
1. I am a closed figure with four straight sides.
2. I have two long sides and two short sides
3. I have a right angle.
4. My two long sides are parallel.
5. I have two right angles.
6. My two long sides are not the same length.
7. My two short sides are not the same length.
8. My two short sides are not parallel.
9. I have only two right angles.

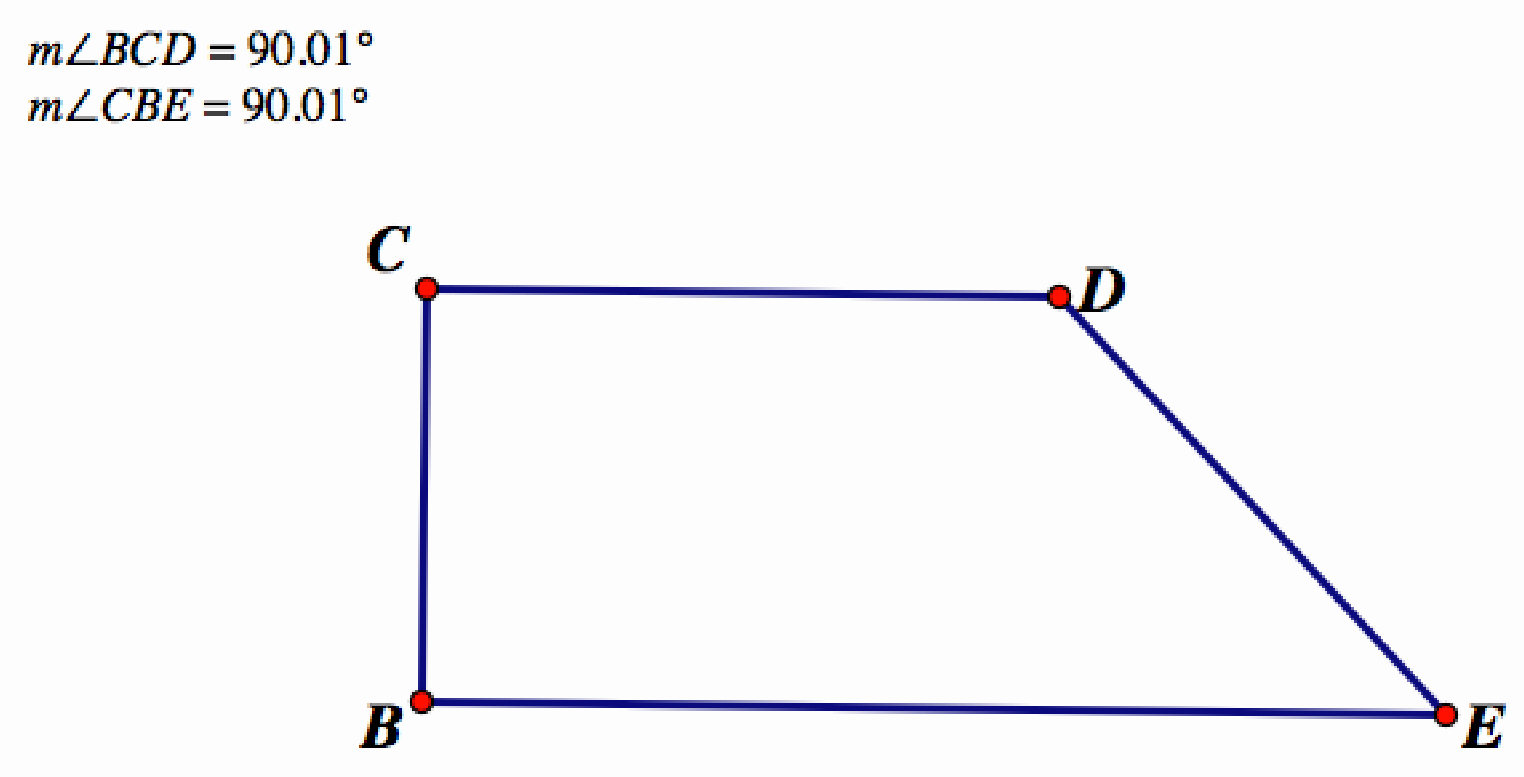
Your solution:

- I am a _________________.
- You knew this at clue number ___________.
- You knew this because____________________

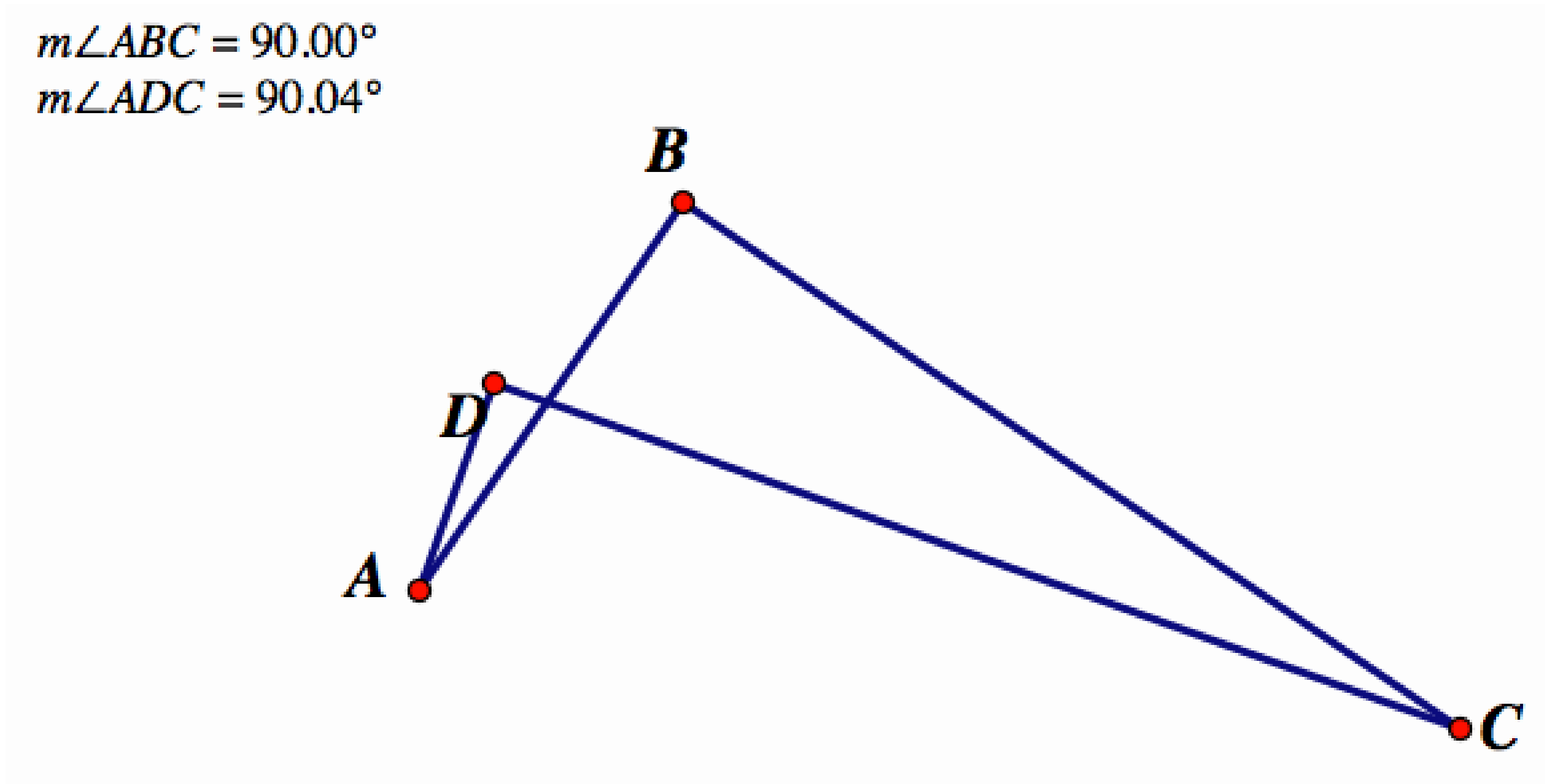
____.

$m\angle ABC = 90.00°$
$m\angle ADC = 90.01°$





$m\angle BCD = 90.01°$
$m\angle CBE = 90.01°$
C
D
B
E


m∠ABC = 90.00°
m∠ADC = 90.04°
B
D
A
C

1. **Younger students often make little or no use of the actual clues—they latch onto a visual prototype.**
2. **Shapes are seldom identified in the minimal number of clues.**
3. **Students often make extra, unwarranted assumptions.**
4. **Older students often use clues as "necessary conditions" to support their guessed shape, rather than as "sufficient conditions to guarantee a shape."**
5. **Students often don't exhibit a lot of confidence in their decision.**

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Research Scenario 3: Reasoning using information from Sampling

Listening to Students Statistical Reasoning: The Case of Sampling Tasks

Work by Mike Shaughnessy, Matt Ciancetta, Dan Canada, and Jennifer Noll (Manuscripts currently under review, some early papers appear in refereed conference proceedings).

Bi-directional nature of sampling tasks

- ➔ If you know the population mix, predict the results for one sample, or for repeated samples:

Ex.54% of voters cast theirs for Obama—so in a sample of 100 voters, we expect…?

Bi-directional nature of sampling tasks

- 🡐 If you are given the results of some samples, predict parameters for the population from which the samples were drawn:

Ex. In a random sample of 100 Bostonians, 61 favored Obama’s position on healthcare? What percent of all Bostonians favor Obama’s position?

Student Experiences with Sampling

In the U.S., it is often the case that students do not have many opportunities to work with sampling, or sampling distributions in their school mathematics.

They may be asked to predict ‘one’ outcome, or find the probability of an outcome, but they are rarely asked to think about what might happen if repeated samples are drawn.

Observation 1

For students to reason and make sense when using sampling distributions in either direction, they need to be strong proportional reasoners!

Observation 2

When reasoning from sampling distributions, students also need to be able to identify and explain trends amidst variability in the data.

Some research questions….

1) How do students reason when given bi-directional tasks involving repeated samples, sample to population, and population to sample?
2) How *do* students reason about the variability in repeated samples of data? Are there developmental paths in students understanding about variability?

Research on the Development of Students' Conceptions of Sampling

Study 1:

- National Science Foundation Project
- 232 Students grades 6 - 12
- 6 Schools- 2 urban, 1 rural, 3 suburban
- 10 Classrooms (6 research & 4 comparison classes); 7 secondary classes, 3 middle school classes

Research on the Development of Students' Conceptions of Sampling

Study 2: Similar study done with college students (Ciancetta, 2007)

- Undergraduates students in beginning stat
- Upper division mathematics students

Study 3: Similar tasks and interviews done with Graduate students who were teaching intro statistics (Noll, 2008)

Data gathered in the project

- Pre-post surveys on sampling tasks
- Videotaped semi-structured task based interviews with students (2M, 2F were randomly selected from each of the six research classes in Study 1)
- Videotapes of classroom teaching episodes

Repeated Sampling. Suppose that you have a very big container with *1000* candies in it, 650 are red, and 350 are yellow. The candies are all mixed up in the container. Students scoop out 100 candies at a time, count the number of reds, and then replace and remix the candies.

a) What would you predict for the numbers of reds in pulls of 100 candies by 6 students?

______ ______

______ ______

______ ______

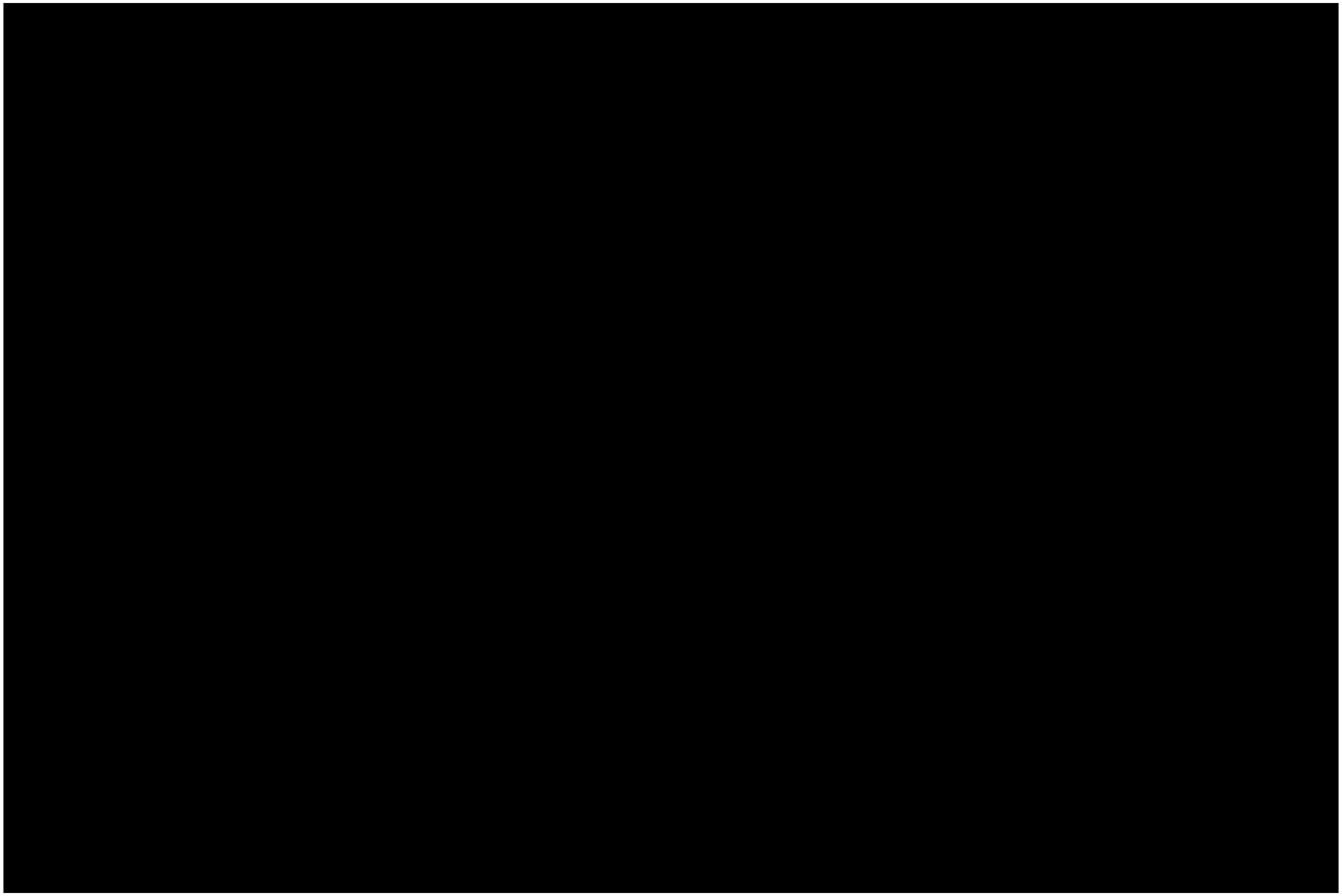
Why do you think this?

b) Suppose that 50 students each pulled out 100 candies, wrote down the number of reds, then put them back and mixed them up for the next person. Of the 50 students, how many do you think would get:

0 to10 reds?________
11 to 20 reds?________
21 to 30 reds?________
31 to 40 reds?________
41 to 50 reds?________
51 to 60 reds?________
61 to 70 reds?________
71 to 80 reds?________
81 to 90 reds?________
91 to 100 reds?________

Why do you think this? Explain your choices.

Student Predictions
Repeated Sampling



“Probability is not Perfect”

- James (G12), from one of the research classes
- Talked about what “should” happen (65 reds each time), but it doesn’t in real life
- Uses multiple aspects of a distribution--both center and shape-- says ‘more of them will be above 65.’

4. **Repeated Sampling.** Suppose that you have a very big container with *1000* candies in it, 650 are red, and 350 are yellow. The candies are all mixed up in the container. Students scoop out 100 candies at a time, count the number of reds, and then replace and remix the candies.

a) What would you predict for the numbers of reds in pulls of 100 candies by 6 students?

75   60

70   51

80   85

Why do you think this?

b) Suppose that 50 students each pulled out 100 candies, from the bowl, wrote down the number of reds, put them back, mixed them up. Of the 50 students, how many of them do you think would get:

0 to10 reds?__1__
11 to 20 reds?__1__
21 to 30 reds?__2__
31 to 40 reds?__2__
41 to 50 reds?__4__
51 to 60 reds?__6__
61 to 70 reds?__10__
71 to 80 reds?__10__
81 to 90 reds?__8__
91 to 100 reds?__7__

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Why do you think this? Explain your choices..

Student Predictions
Repeated Sampling

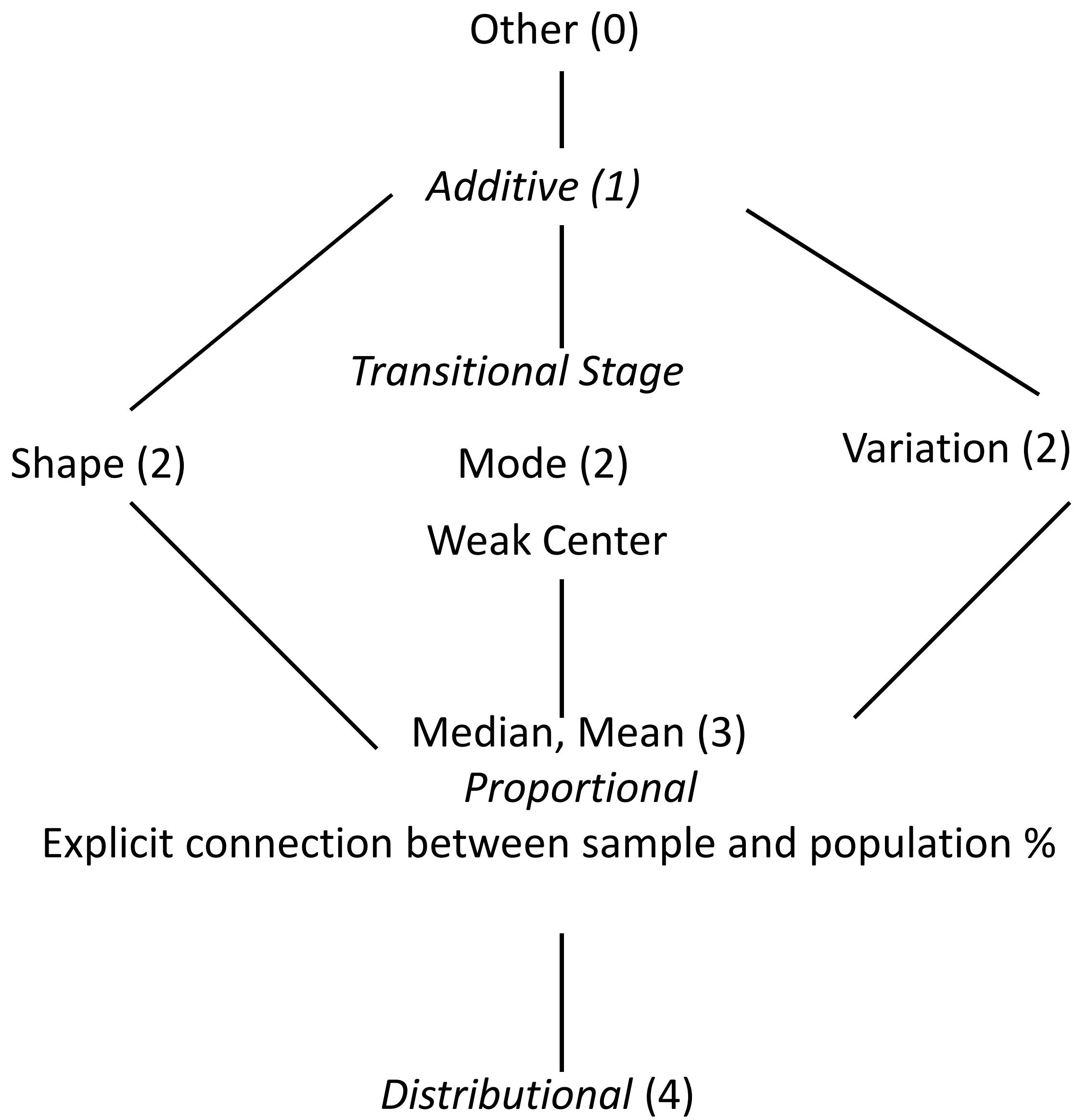
“Because there are more reds!”

- Keosha (G10), a research class student, stuck to her ‘more reds’ argument
- Interviewer pushes for “why 65, why not 75 or 85?”, she insists “65 because there are more reds”
- While no *explicit* use of proportional reasoning is verbalized, the seeds of it may be there

Lattice of Development in Students' Distributional Thinking

- The Aspects of Distribution—Center, Shape, and Variation—develop within student thinking over time
- There appear to be some identifiable *stages* in the development of student thinking about distributions, additive, proportional, distributional

Stages of Reasoning about Repeated Sampling



Other Things we learned from student thinking and reasoning about sampling distributions

- Even seemingly straightforward tasks elicit a wide spectrum of student thinking
- Students' initial comments are not always an accurate indication of what they know
- Multiple interviews with the same student provide a more reliable picture of their thinking

Implications of researching our students' thinking for Instructors

- We need to *systematically* listen to our students' reasoning as it can provide valuable information about the development of their thinking

- Knowing how are students are thinking can inform us on what our next instructional steps might be to help them grow

THANK YOU

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