MATH 101

FUNCTIONS, LINEAR, & QUADRATIC FUNCTIONS

SPRING 2009

Members of Group:

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Remember, A **Function** is a correspondence between an input variable and an output variable such that each input variable has exactly one output variable.

For example, one can say " **The revenue, R,** of a local movie theater is a **function of the number of people, n,** in attendance." It is a function, because for each person in attendance there is a unique revenue generated by the theater.

Using function notation this relationship can be written as
$R = f(n)$; f is the name of the function . Although parentheses may remind you of multiplication, there is NO multiplication in this notation. It is simply read as " R equals f of n". The input variable is always inside the parentheses and the output variable is always alone on the other side of the equal sign. Hence **"output variable is a function of the input variable."**

PART 1

Example: Suppose a theater charges \$6.00 for admission for a movie at any given time for an adult or a child. A table to illustrate this situation is the following.

n	$R = f(n)$
0	*0*
1	6
2	12
3	18
4	24

As you can see, for each input there is exactly one output. Hence, Revenue, R, is a function of the number of people, n, in attendance. The sentence, " The movie theater generates \$12 if two patrons attended" can be written as $f(2) = 12$ using function notation.

Problem 1: *" **The amount of money Bruce makes a week*** is **a function of *the number of hours he works".***

a) In the above sentence which is the input variable and which is the output variable?

b) Choose variable names for the input variable and the output variable and write using function notation.

c) Suppose Bruce made $180 working 24 hours this past week. Using your variables write this situation using function notation.

Problem 2: ***"The population of a Country*** *is* **a function of *the year."*** If $t =$ number of years since 2000 and $P =$ population in millions, then $P = f(t)$.

a) Write a sentence interpreting the meaning of $f(5) = 12.5$ in the context of this situation.

b) Write the phrase " In which year would the population of the Country be three million people?" using function notation.

c) Write a sentence interpreting the meaning of $P = f(a)$

Problem 3: Let $m(t)$ represent the number of qualified Doctors working in Canada in thousands t years after 1968. Write the practical meaning of each of the following statements.

a) $m(30) = 112$

b) $12 = m(t)$

c) $m(38) = m(40)$

d) $m(-12) = 12$

Example: The daily demand, n, for the number of DVD's sold at a store is a function of the price, p in dollars. Using function notation this can be written as $n = f(p)$. At a price of \$$p$ for a DVD a quantity of n are demanded daily.

$f(7) = 50$ interprets as "at a price of \$7 per DVD, 50 DVD's are demanded daily".

A change inside the parentheses can be generally referred to as an inside change. Consider

$n = f(p+2)$ means the number of DVD's demanded daily at a price of \$2 more than \$$p$ for a DVD.

A change outside the parentheses can be generally referred to as an outside change. Consider

$n+2 = f(p)$ means two more DVD's are demanded daily than n, at a price of \$$p$ per DVD.

Problem 4: Sri Lanka is an island nation off the coast of South India. The city of Jaffna and Kandy have a rich history and tradition among its two major ethnic groups Tamils and Singhalese respectively.

Let $J(t)$ be the population in thousands of the city of Jaffna t years after 1970. Let $K(t)$ be the population in thousands of the city of Kandy t years after 1970. Both populations are related by $K(t) = J(t-2)+3$

a) Write a sentence describing the meaning of $K(0) = 650$

b) Write a sentence describing the meaning of $J(4)$

c) Write a sentence describing the meaning of $J(t+2)$

d) Write a sentence describing the meaning of $K(t)+2$

e) Write in words the relationship between the two populations.

f) Suppose in 1976 the population of Jaffna was 700 thousand, what can you say about the population of Kandy?

Part II

Problem 5: At the closing bell On July 31, 2006, a stock of Company INTC cost $21. The stock of SNDK company cost $48. Monitoring the markets for a week you have noticed Company SNDK gaining approximately $0.30 everyday and during the same time the Stock of INTC has been increasing at a rate of about $1.20 each day. Watching the program "MAD MONEY" on CNBC you are convinced that these patterns are going to continue for some period of time.

a) Write an equation that predicts the price of a stock of Company INTC, *y*, as a function of the day, *x*, since July 31, 2006.

b) Write an equation that predicts the price of a stock of Company SNDK, *y*, as a function of the day, *x*, since July 31, 2006

c) Would there be a day where both stocks could end up at the same price? If so when would it happen?

d) Satisfied with the projections of these stocks, you decided to buy $800 worth of INTC stock and SNDK stock on July 31, 2006. Let *z* be the amount of INTC stock and let *a* be the amount of SNDK stock. Find a linear equation that relates the values of *z and a*.

(Help: The roles of the output variable and input variables are not necessarily fixed. For example $C = (F - 32) \cdot \frac{5}{9}$ **gives the relationship between degree Celsius and degree Fahrenheit. In this equation degree Fahrenheit is the input variable and degree Celsius is the output variable. Likewise solving for *F* gives** $F = \frac{9}{5}C + 32$**. In this equation the input variable is degree Celsius and the output variable is degree Fahrenheit.)**

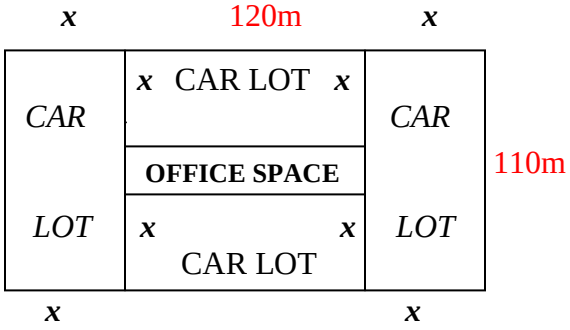
e) Find the slope of this equation and interpret its meaning in the context of the problem.

f) Find the vertical intercept of this equation and interpret its meaning in the context of the problem.

g) Find the horizontal intercept and interpret its meaning in the context of the problem.

Part III

Problem 6:
Your group is in charge of designing an AUDI dealership in town. The dealership owns a property that is rectangular in shape, and measures 120 meters wide and 110 meters long. The dealership estimates they would need an area of $6000m^2$ for their car lot to accommodate all the vehicles that they receive at any given time. For customers to easily navigate around the lot and the display of cars to be visible by the public that goes by the roads each day they request their office space to be in the middle of the property. They prefer a uniform width around its perimeter.



a) Suppose you chose the letter x to represent the width of the uniform border that separates the car lot and the office space, what is the width of the lot?

b) What will be the length of the uniform border in terms of x?

Recall, the **domain** of a function is all the meaningful replacements for the input variable.

The practical domain or realistic domain can be defined as the meaningful replacements for the input variable that makes sense in the context of the problem. For example in the equation $y = 4x$, the domain is all real numbers. Since you can plug in any real number for x and the function is still defined.

Now consider, $P = 4s$, where P is the perimeter of a square and S is the length of a side of the square. The practical domain in this situation becomes $0 < s \leq s_{\max}$. Notice s is not negative or zero because, if either were true there is no square to begin with.

Now continue with the rest of the questions.

c) What will be the practical domain for x in this situation? That is, What can be the smallest and largest values of x that would make practical sense in this specific problem.

d) What will be the area of this car lot in terms of x? Write using function notation.

e) The lot is required to have an area of $6000\,m^2$. What equation would you set up in terms of x to include that information?

f) Solve the equation using any algebraic method of your choice and approximate results to two decimal places.

g) How many answers did you obtain? Which answer is within the practical domain?

h) What are the dimensions of the car lot?

i) Suppose the dealership estimates all they need for office space is $2000\,m^2$, what value of x will provide the maximum area for the car lot?