

Average Rate of Change & Slope of a Line
MATH 092

Average Rate of Change

Functions are used to model the way one quantity changes with respect to another quantity. For instance, how does the distance traveled change as time changes from 1 hour to 4 hours? Or how does profit change as the number of items sold changes from 3000 to 4500? To help us study the way quantities change we use a special symbol, Δ (delta). The symbol Δ is used to represent the phrase “change in.” The precise mathematical definition of a Function will be given later in chapter 3.

The following table shows Eric’s weight over a five week diet program.

Weeks, w	0	1	2	3	4	5
Weight, P (pounds)	190	184	181	184	184.5	179

1. Find the average rate of change of Eric’s weight over the first three weeks (i.e. $w = 0$ to $w = 3$)

Solution: The **average rate of change of a function over a given input interval is**

$$\frac{\text{change in output}}{\text{change in input}}$$

We write : $\frac{\Delta P}{\Delta W} = \frac{184 - 190 \text{ pounds}}{3 - 0 \text{ weeks}} = \frac{-6}{3} = -2 \text{ pounds/week}$

Practical Meaning: Eric **lost** an average of _____ during the first 3 weeks.

2. Find the average rate of change of Eric’s weight from week 2 to week 4 (i.e. from $w = 2$ to $w = 4$)

$$\frac{\Delta P}{\Delta W} =$$

Practical Meaning: Eric _____ an average of _____ from week 2 to week 4.

Definition: The **average rate of change of a function** over a given input interval is

$\frac{\text{change in output}}{\text{change in input}}$. The rate of change is a number that indicates how much and in what direction the output changes when the input changes by one unit.

Note: Average rate of change numbers have units of measure such as “miles per gallon,” “cost per hour.” If you write the correct word labels in the numerator and in the denominator then your average rate of change label will always be in the form: numerator units per 1 denominator unit.

Example:

Suppose the output is cost in dollars, C , and the input is pounds, p . If 6 pounds cost \$18 and 2 pounds cost \$6, then the average rate of change is

$$\frac{\Delta C}{\Delta P} = \frac{18 - 6 \text{ dollars}}{6 - 2 \text{ pounds}} = 3 \text{ dollars per pound or } \$3/\text{lb}$$

You have decided to buy a new Honda Accord LX, but you are concerned about the value of the car depreciating over time. You search the internet and obtain the following information at [www. Internetautoguide.com](http://www.Internetautoguide.com)

2008 Accord LX

- Suggested retail price \$21,000
- Depreciation per year \$1,200 (assume constant)

1.a. Complete the following table in which v represents the value of the car after n years of ownership.

n , Years	v , Value in Dollars
0	
1	
2	
3	
5	
8	

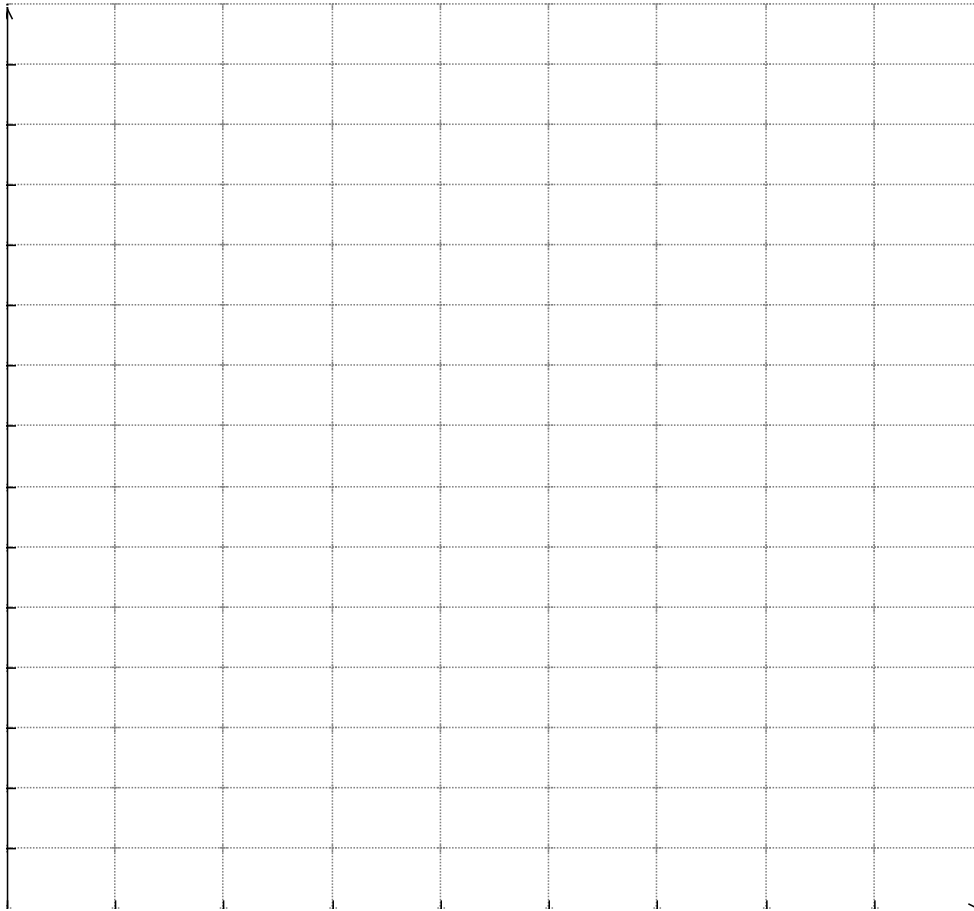
- 2.a. Select two ordered pairs of the form (n, v) from the table in problem 1 and determine the average rate of change.
- b. What are the units of measure of the average rate of change?
- c. What is the practical meaning of the sign of the average rate of change?
- d. Select two different ordered pairs and compute the average rate of change.
- e. Select two ordered pairs not used in parts a or d, and compute the average rate of change.
- f. Using the results in parts a, d, and e, what can you infer about the average rate of change over any interval of time?

If the computation of the average rate of change using any two ordered pairs yields the same result, the average rate of change is said to be constant.

Definition

When the average rate of change, $\frac{\text{change in output}}{\text{change in input}}$, remains a **constant** it is said to be **linear**.

- g. Using the ordered pairs in the table, plot points and draw the graph in the grid attached, Please label axes and scale properly.



Recall, the graph of every linear equation is a straight line. The constant average rate of change is called the slope of the line and is denoted by the letter m

Definition

If x represents the input variable and y represents the output variable, then the slope m is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ if } x_1 \neq x_2. \text{ i.e } m \text{ is the slope of the line through the points}$$

$$(x_1, y_1) \text{ and } (x_2, y_2). \text{ Geometrically, } m = \frac{\text{rise}}{\text{run}}$$

- 5.a. What is the slope of the line graphed in problem 4

- b. What is the relationship between the slope of the line and the average rate of change?
- c. What is the practical meaning of the slope in this situation?
6. Find the slope of each line through the given points.
- a. Through $(6, -4)$ and $(5, 4)$ b. Through $(10, -2)$ and $(7, -8)$
- c. Through $(2, -3)$ and $(6, -6)$ d. Through $(7, -2)$ and $(6, -2)$
- e. Through $(-2, 5)$ and $(-2, 4)$

A line with positive slope rises from left to right.

A line with negative slope falls from left to right.

Horizontal lines, which have equations of the form $y = k$, have **zero** slope.

Vertical lines, which have equations of the form $x = k$, have **undefined slope**.

Vertical Intercept

Definition

The vertical intercept is the point where the graph crosses, or intercepts, the vertical axis. The input value of a vertical intercept is always zero. If the output variable is represented by y , the vertical intercept is referred to as the y -intercept.

c. $y - 3 = -2$

d. $x + 5 = -2$

Horizontal Intercepts

Definition

A horizontal intercept of a graph is a point where the graph meets or crosses the horizontal axis. The output value of the horizontal intercept is always zero. If the input variable is represented by x , the horizontal intercept is referred to as the x -intercept.

10.a. Determine the horizontal intercept (n -intercept) of the graph of the car value equation

b. What is the practical significance of the horizontal intercept? Include units.

Suppose your best friend decides on a Honda Civic LX with a suggested retail price of \$17,500 that depreciates \$1200 yearly.

11. Complete the following table in which V represents the value of the car after n years of ownership.

n , Years	v , Value in Dollars
0	
1	
2	
3	
5	
8	

12. Use the ordered pairs above plot points and draw the graph in the grid given in 2g.

13. Do the graphs appear to be similar in any way? If yes, what do you notice?

14. Write an equation for v in terms of n for the table in #11.

15. Write the vertical intercept and the slope from this equation.

16. What is the connection between the slopes from your car equation and your friends' car equation?

Two lines are parallel if they have the same slope and different vertical intercepts.

Two lines are perpendicular when the product of their slopes is -1. i.e $m_1 = -\frac{1}{m_2}$

17. Decide whether each pair of lines is parallel, perpendicular, or neither.

a) $x + y = 2$
 $-x - y = -1$

b) $3x - 7y = 35$
 $7x - 3y = -6$

c) $8x - 9y = 6$
 $8x + 6y = -5$

d) $2x + 3y = -1$
 $3x - 2y = 9$