

---

## P R E F A C E

**M**athematics programs at two-year and four-year colleges as well as at many universities serve students from diverse personal and academic backgrounds who begin their postsecondary educations with a wide variety of educational goals and personal aspirations. In addition to serving students who are prepared to study calculus, these mathematics programs must accommodate students who intend to study calculus but enter college unprepared to do so. They must also serve students who do not intend to study calculus. The career aspirations of some of these students are such that requirements for graduation and for job placement can be satisfied through the study of mathematics below the level of calculus. Mathematics taught at this level in two-year colleges and in the lower division of four-year colleges and universities is referred to in this document as "introductory college mathematics." This phrase will be used to include college algebra, trigonometry, introductory statistics, finite mathematics, and precalculus, as well as all courses presently characterized as developmental mathematics. Noncalculus-based mathematics courses for technical or occupational programs and mathematics courses for elementary teachers and those for liberal arts majors are also considered part of introductory college mathematics for the purposes of this document.

Introductory college mathematics constitutes a large percentage of the offerings at postsecondary institutions. The survey done in 1990 by the Conference Board of the Mathematical Sciences (Albers, Loftsgaarden, Rung, & Watkins, 1992) revealed the following data concerning students studying introductory college mathematics (enrollment in computer science is not considered in these data):

Of 1,295,000 students studying mathematics in two-year college mathematics departments,

- 724,000 (56%) were studying at the remedial level
- 245,000 (19%) were studying precalculus
- 17,000 were studying technical mathematics with no calculus prerequisite
- 35,000 were studying mathematics for liberal arts
- 9,000 were studying mathematics for elementary teachers

In addition, approximately another 126,000 students were studying introductory mathematics in two-year colleges in departments other than mathematics departments.

In four-year college and university mathematics departments,

- 261,000 (15% of the mathematics enrollment) were studying at the remedial level
- 593,000 (34% of the mathematics enrollment) were studying precalculus

These statistics indicate that introductory mathematics courses serve the needs of more than half the students studying mathematics in college.

The need for change in mathematics education has been documented in several national reports issued in the past decade, and significant change has begun at several levels. The *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 1989) presents comprehensive recommendations for innovative approaches to curriculum and pedagogy for kindergarten through twelfth grade; *A Curriculum in Flux* (Davis, 1989) makes specific recommendations for the curriculum at two-year colleges; *Reshaping College Mathematics* (Steen, 1989) outlines a proposed undergraduate curriculum; *Moving Beyond Myths* (National Research Council [NRC], 1991) calls for dramatic changes to "revitalize" undergraduate education; and *Everybody Counts* (NRC, 1989) makes specific recommendations for changes in mathematics programs from kindergarten through graduate school. Furthermore, calculus instruction has been reformed at many institutions [see Crocker (1990) for a description of the development of calculus reform, Ross (1994) for a description of recent reform initiatives, and Tucker and Leitzel (1994) for an assessment of calculus reform].

Until now no group has attempted to establish standards for mathematics programs that specifically address the needs of college students who plan to pursue careers that do not depend on knowledge of calculus or upper-division

mathematics, or those students who need calculus but enter college unprepared for mathematics at that level. Almost all postsecondary institutions offer introductory mathematics courses, but in two-year colleges these courses constitute over 80 percent of the offerings (Albers et al., 1992). The American Mathematical Association of Two-Year Colleges (AMATYC) is the organization whose primary mission includes the development and implementation of curricular, pedagogical, assessment, and professional standards for mathematics in the first two years of college. In this document, AMATYC, with assistance from other professional mathematics organizations, has undertaken the challenge of setting standards for curriculum and pedagogy in introductory college mathematics.

Building upon the reform efforts cited above this document presents standards that are designed for adult students, many of whom are underprepared for the study of college-level mathematics. The purpose of *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* is to address the special circumstances of, establish standards for, and make recommendations about two-year college and lower-division mathematics programs below the level of calculus.

Three sets of standards for introductory college mathematics are defined in Chapter 2.

The **Standards for Intellectual Development** address desired modes of student thinking and represent goals for student outcomes. They include

- Problem Solving
- Modeling
- Reasoning
- Connecting with Other Disciplines
- Communicating
- Using Technology
- Developing Mathematical Power

The **Standards for Content** provide guidelines for the selection of content that will be taught at the introductory level. They include

- Number Sense
- Symbolism and Algebra
- Geometry
- Function
- Discrete Mathematics
- Probability and Statistics
- Deductive Proof

The **Standards for Pedagogy** recommend the use of instructional strategies that provide for student activity and interaction and for student-constructed knowledge. They include

- Teaching with Technology
- Interactive and Collaborative Learning
- Connecting with Other Experiences
- Multiple Approaches
- Experiencing Mathematics

The chapters that follow interpret the standards in various program areas, discuss the implications of the standards in several areas of mathematics education, and provide the design for establishing a nationwide effort to disseminate and implement the standards. Illustrative examples of problems aimed at capturing the vision and spirit of the standards appear in the Appendix.

The standards included in this document reflect many of the same principles found in school reform [for example, see NCTM (1989)] and calculus reform [see Crocker (1990), Ross (1994), and Tucker and Leitzel (1994)]. However, they differ in some respects and focus on the needs and experiences of college students studying introductory mathematics. In particular,

- **The Foundation** includes topics traditionally taught in "developmental mathematics" but also brings in additional topics that all students must understand and be able to use. Courses at this level should not simply be repeats of those offered in high school. Arithmetic, algebra, geometry, discrete mathematics, probability, and statistical concepts should be integrated into an in-depth applications-driven curriculum. The goal of this curriculum is to expand the educational and career options for all underprepared students.

- **Technical Programs** place strong emphasis on mathematics in the context of real applications. The mathematics involved is beyond the level of sophistication experienced in the Foundation. Mathematics faculty, in cooperation with their colleagues in technical areas or with outside practitioners, should select content that prepares students for

the immediate needs of employment. However, at the same time, students should learn to appreciate mathematics and to use mathematics to solve problems in a variety of fields so that they will be able to adapt to change in their career and educational goals.

• **The Mathematics-Intensive, Liberal Arts, and Prospective Teachers Programs** place heavy emphasis on using technology, developing general strategies for solving real-world problems, and actively involving students in the learning process. Students in each of these programs are either pursuing bachelor's degrees or intending to pursue bachelor's degrees after completing their associate's degrees. Introductory college mathematics is intended to provide the needed prerequisite knowledge for further study of mathematics or for courses in other disciplines that require a *knowledge of mathematics at the introductory level*. At the same time, liberal arts majors and prospective elementary school teachers should gain an appreciation for the roles that mathematics will play in their education, in their careers, and in their personal lives.

## COVER DESIGN

The cover art, created by graphics artist Karen Meyer Rappaport, depicts two crossing roads. These roads serve as a metaphor for two intersecting national trends: a growing societal need for a citizenry with a sophisticated level of mathematical preparation and an increasing number of academically underprepared students seeking entrance to postsecondary education. The cube situated at the intersection represents the three dimensions of the standards – intellectual development, content, and pedagogy – that are intended to guide mathematics faculty along the way to reform.

## VIGNETTES AND MARGIN NOTES

The vignettes that appear throughout the document are true stories that help give a human face to the issues the report raises. The margin notes are of two types: excerpts that emphasize key ideas, and quotes from experienced mathematics educators and students that support the views expressed in the document. The quotes were obtained from speeches, published materials, position papers, document reviews, and student reports. The unidentified quotes may be attributed to the Task Force.

## ACKNOWLEDGMENTS

Each of the groups listed in the front matter played an important role in the planning and development of this document. The work of the Planning Group led to the development of the project proposal. The National Steering Committee developed principles to guide the writing of the document. The Task Force, aided by the consultants, began development of the document at a week-long writing session in Memphis in June 1993. The Task Force chose some of its members to serve as a writing team to receive feedback on the draft and revise the document accordingly. The Advisory Panel members reviewed the final draft and offered suggestions for final revisions. We wish to thank the numerous reviewers whose perspectives were considered at each stage of development. A special thank you is extended to Daniel Alexander, Michael Davidson, Sheldon Gordon, John Jenkins, and Jack Rotman, who submitted position papers, quotes from which appear as margin notes. The Task Force wishes to thank Allyn Jackson for helping edit the final draft, and artist Karen Meyer Rappaport for designing the cover. We thank Addison-Wesley Publishing Company for contributing the artwork and Prentice Hall, Inc. for donating the page designs and copy editing.

This document is intended to stimulate faculty to reform introductory college mathematics before calculus. These standards are not meant to be the "final word." Rather, they are a starting point for your actions.

Don Cohen  
Editor

---

## FOREWORD

**C***rossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* has two major goals: to improve mathematics education at two-year colleges and at the lower division of four-year colleges and universities and to encourage more students to study mathematics. The document presents standards that are intended to revitalize the mathematics curriculum preceding calculus and to stimulate changes in instructional methods so that students will be engaged as active learners in worthwhile mathematical tasks. In addition, the implications of these changes in such areas as faculty development and student assessment are discussed. Preparation of these standards has been guided by the principle that faculty must help their students think critically, learn how to learn, and find motivation for the study of mathematics in appreciation of its power and usefulness.

This document represents a major effort of the American Mathematical Association of Two-Year Colleges (AMATYC), assisted by representatives of many other national mathematics education organizations. AMATYC's previous efforts to improve mathematics education at the two-year college level have taken the form of development of policy statements and guidelines. The most notable recent efforts have been *Guidelines for the Academic Preparation of Mathematics Faculty at Two-Year Colleges* (AMATYC, 1992) and *Guidelines for Mathematics Departments at Two-Year Colleges* (AMATYC, 1993). AMATYC was inspired toward additional efforts in education reform when in August 1991 the Mathematical Sciences Education Board (MSEB) convened a small planning group at the National Academy of Sciences to discuss two-year college mathematics education. Several members of the AMATYC leadership met with representatives from MSEB and others interested in two-year college mathematics education. This meeting and the subsequent efforts of the representatives of MSEB served to focus and launch the initiative.

The work of the Planning Group led to the development of a multiple-phase proposal designed to address curriculum and pedagogy reform initiatives at two-year colleges. The Exxon Education Foundation provided funds to assist with the initiative. AMATYC also committed funds while at the same time seeking additional support from other sources. Subsequently, the focus of the initiative was broadened to include all lower-division mathematics education below calculus, and substantial funding was received from the National Science Foundation (NSF). Marilyn Mays was designated Principal Investigator, with Karen Sharp and Dale Ewen as Co-Principal Investigators. A steering committee was formed with representatives from AMATYC, the American Mathematical Society (AMS), the Mathematical Association of America (MAA), MSEB, the National Association for Developmental Education (NADE), and the National Council of Teachers of Mathematics (NCTM). This group met in February 1993, in Washington, D.C., to plan the process for the development of a standards document.

The Steering Committee determined that the following principles should guide the deliberations of the Task Force:

- Fundamental changes are occurring in mathematics education as a result of the impact of several national documents, calculus reform efforts, technological advances, and research into how students learn.
- Colleges and universities must prepare an increasingly diverse group of students for further study and the world of work.
- While two-year college and lower-division mathematics students are preparing for a multitude of future occupations, there exists a common core of mathematical experiences, viewpoints, concepts, and skills that should be learned by all students.
- The manner in which students learn is inseparable from the content.
- Research regarding how students learn mandates development of new pedagogical methods and implementation of proven teaching techniques.
- The impact of technology both as a mode of instructional delivery and as a mathematical tool requires a redefinition of the mathematics curriculum.
- The demands of the workplace require that all students become empowered citizens capable of critical thinking.

The Steering Committee appointed a Task Force to develop a standards document. The Task Force was asked to formulate recommendations for a foundation program that would serve all students lacking preparation for college-level mathematics courses and to examine the special needs of students in technical, mathematics-intensive, liberal arts, and teacher preparation programs.

Before meeting, members of the Task Force reviewed a large collection of documents and articles on mathematics education reform and wrote position statements detailing their visions of the curricular and pedagogical reforms needed. These statements were distributed to the Task Force prior to the meeting.

The Task Force shaped a common vision for its work at its June 1993 meeting at State Technical Institute at Memphis. They also developed sections of the first draft of this document, originally titled *Standards for Curriculum and Pedagogical Reform in Two-Year College and Lower Division Mathematics*. Those sections were refined in subsequent weeks by the participants. Editor Don Cohen compiled them into a common format and produced a noncirculating draft which went back to the Task Force, to the National Steering Committee, and to a few reviewers and leaders of the mathematics education community. Then the document was further refined and published as a circulating draft. This draft was mailed to all AMATYC members and has been widely distributed outside of the two-year college community. Hearings were held at several national, regional, and state conferences. Reviews were solicited.

A writing team selected from the Task Force met in Dallas, February 24-27, 1994, to consider all of the comments made on the first circulating draft and to start the revision process. A second circulating draft was prepared and sent to the Task Force and other selected reviewers on July 15, 1994. A preliminary final draft was published in October 1994. It was distributed and discussed at several regional and national conferences including the November 1994 AMATYC conference in Tulsa, the January 1995 AMS-MAA joint mathematics meetings in San Francisco, the February 1995 NADE conference in Chicago, and the April 1995 NCTM conference in Boston. *Crossroads in Mathematics* is based on the many comments and reviews that were received.

The members of the Task Force, along with the names of members of the Writing Team, Planning Group, Steering Committee, and Advisory Panel are listed in the front matter.

We are grateful to the National Science Foundation, the Exxon Education Foundation, and Texas Instruments Incorporated as well as to all who contributed to the development and publication of this document.

Marilyn Mays  
Project Director  
AMATYC President  
(1993-95)

Karen Sharp  
Project Co-Director  
AMATYC President  
(1991-93)

Dale Ewen  
Project Co-Director  
AMATYC President  
(1989-91)

